



# Robotics

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# Goal for this course

- **Design: soft hand design x1**
- **Perception: vision, point cloud, tactile, force/torque x1**
- **Planning: sampling-based, optimization-based, learning-based x3**
- **Control: feedback, multi-modal x2**
- **Learning: imitation learning, RL x2**
- **Simulation tool (pybullet, matlab, OpenRAVE, Issac Nvidia, Gazebo)**
- **How to get a robot moving!**

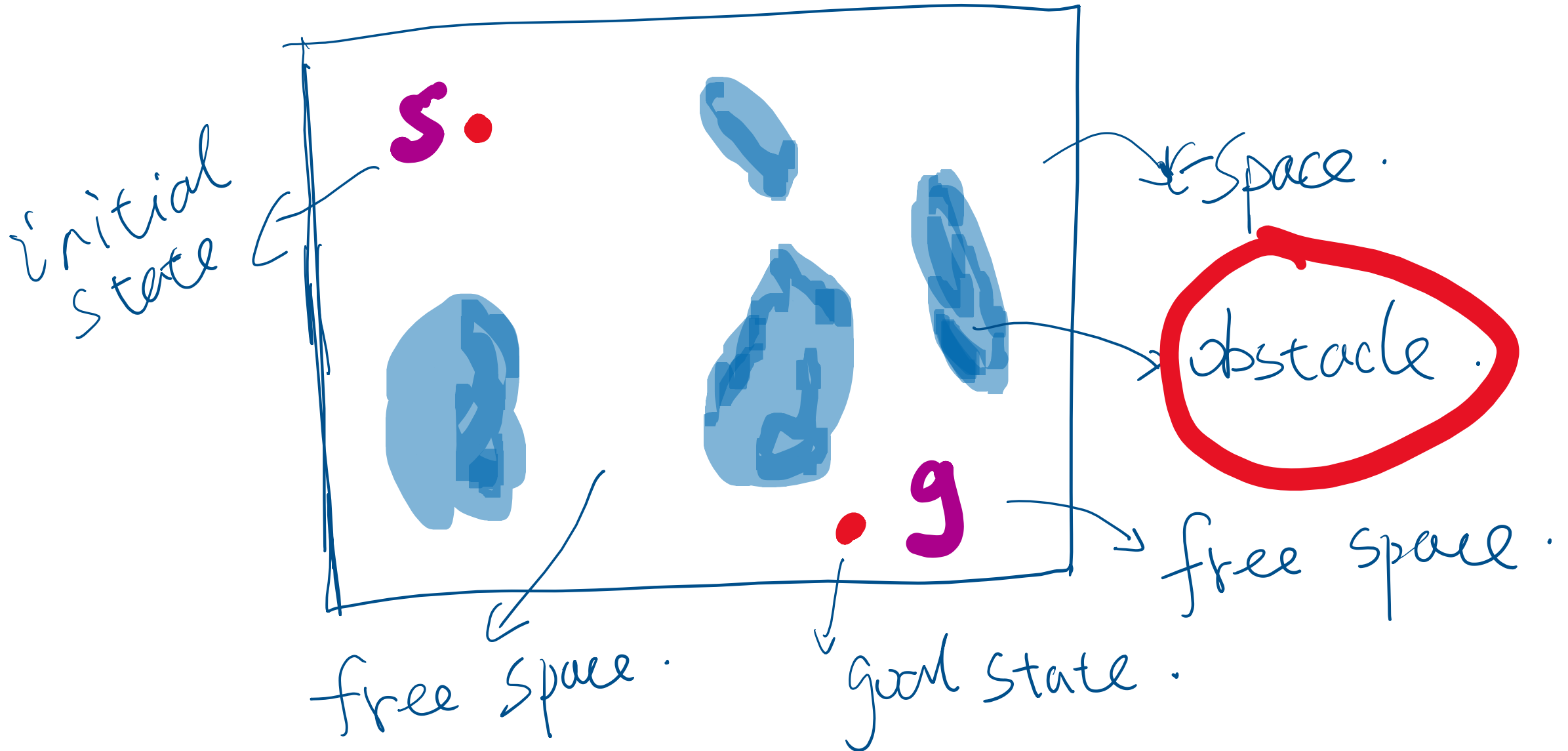


# Today's Agenda

- **Recap of sampling-based approach (~10)**
- **Recap of optimization-based approach (~20)**
- **Drawback of sampling and optimization (~5)**
- **Recap of perception-action loop (~2)**
- **Learning-based motion planning (~5)**
- **Imitation learning (~20)**
- **Reinforcement learning (~10)**

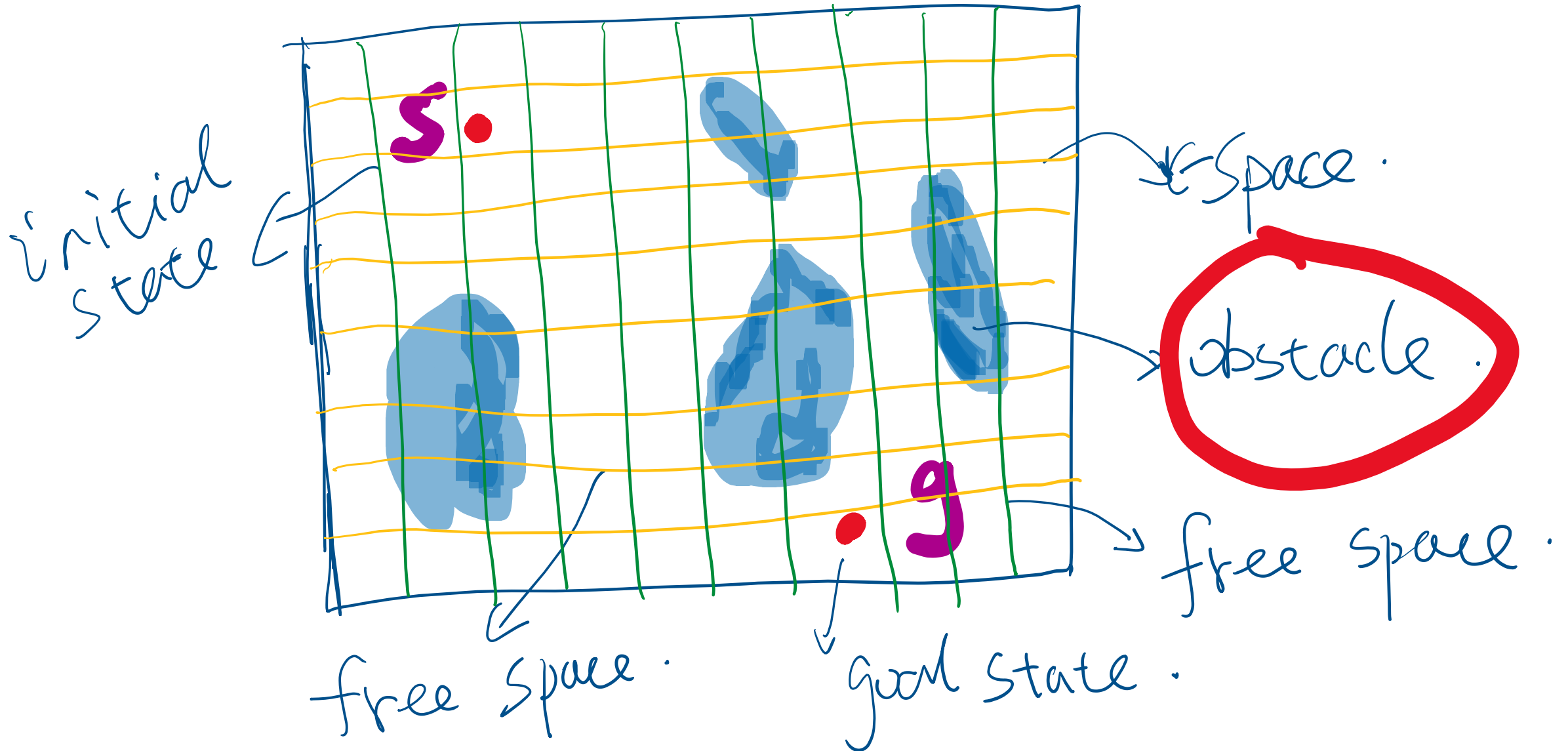


# Motion Planning in 2D





# Motion Planning in Grid World



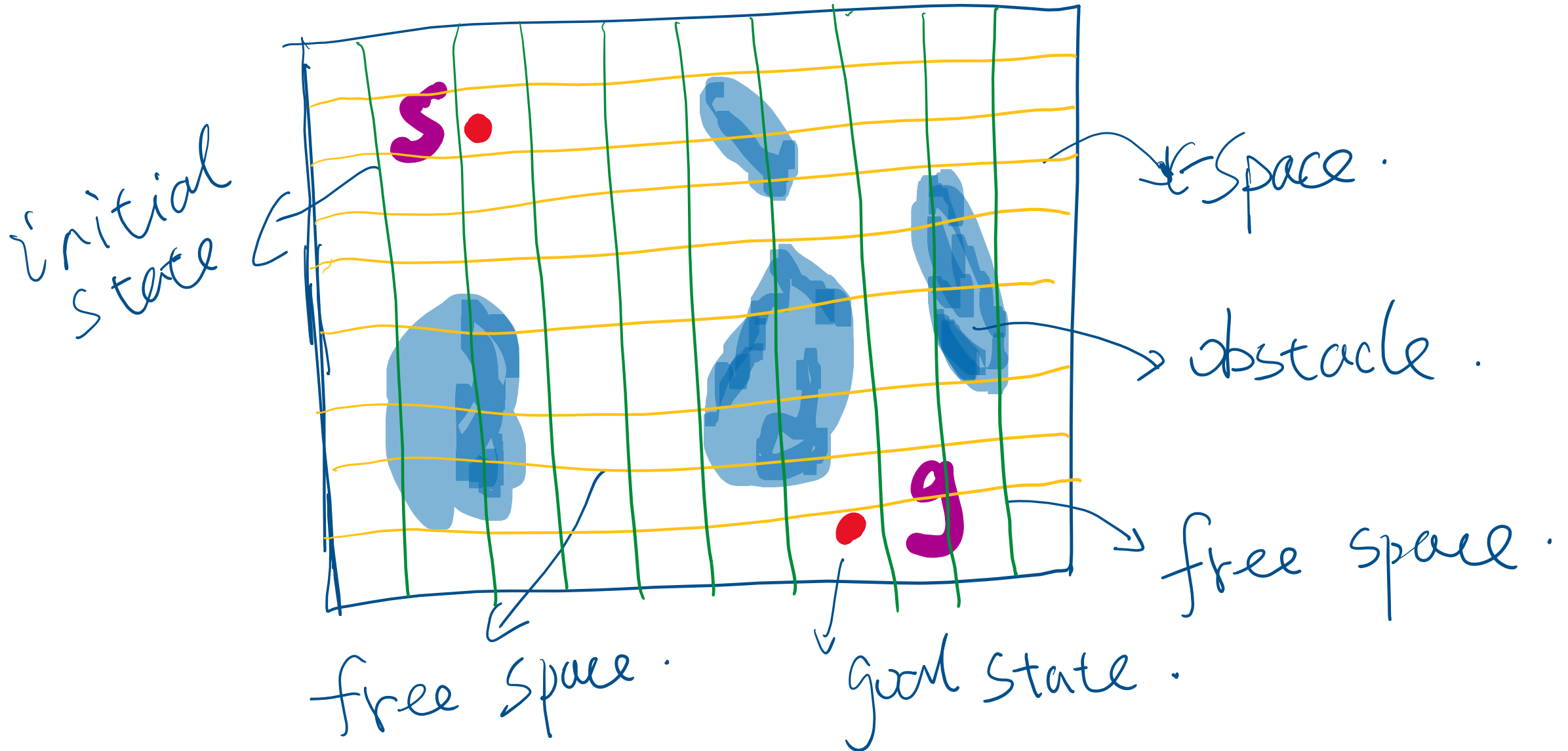


# Recap of sampling-based approach

- **Completely describing and optimally exploring is too hard in high dimension space**
- **It is not necessary**
- **Limit ourselves to finding a “good” sampling**



# Sampling

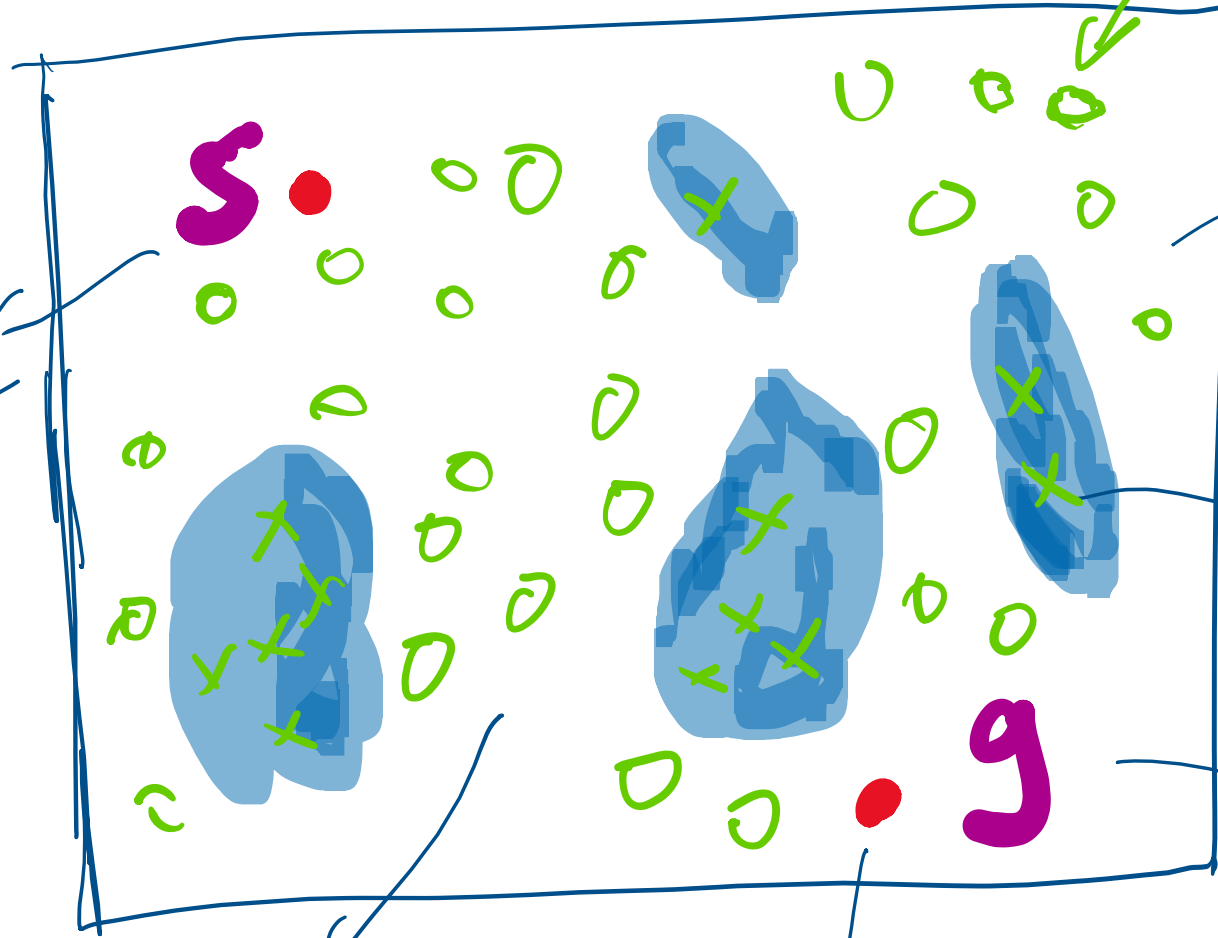




# Sampling

sample random locations

Initial State



Space.

obstacle.

free space.

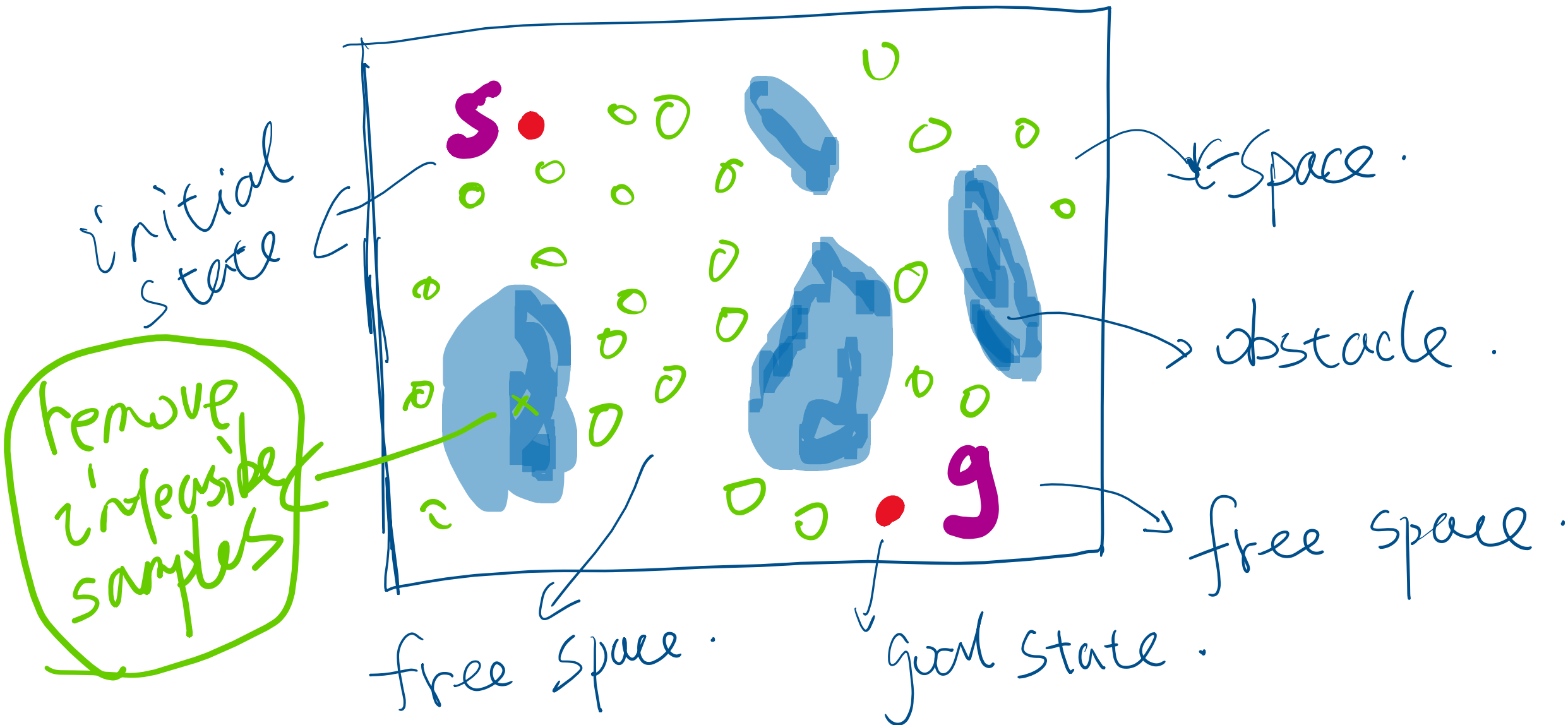
free space.

Goal State.



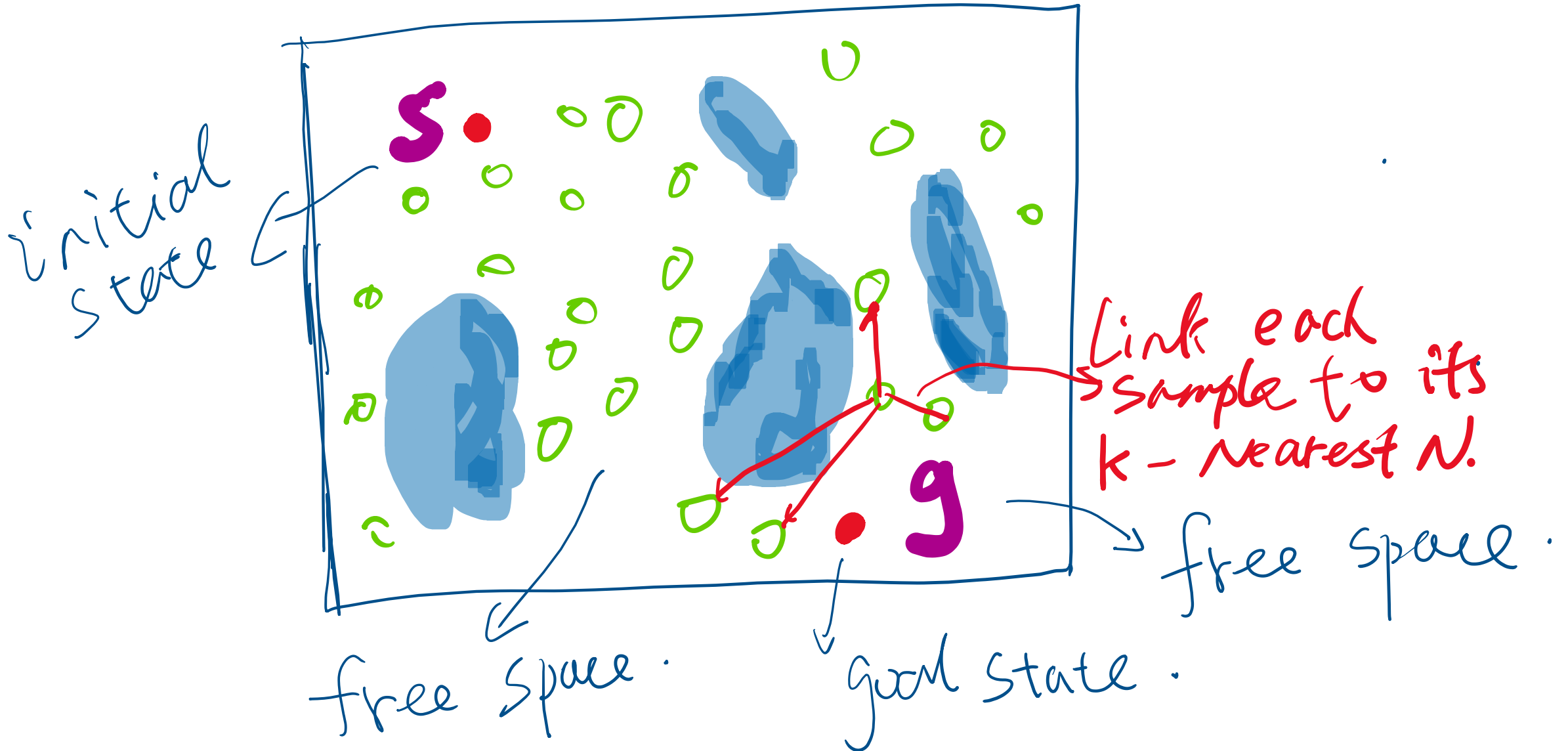


# Sampling



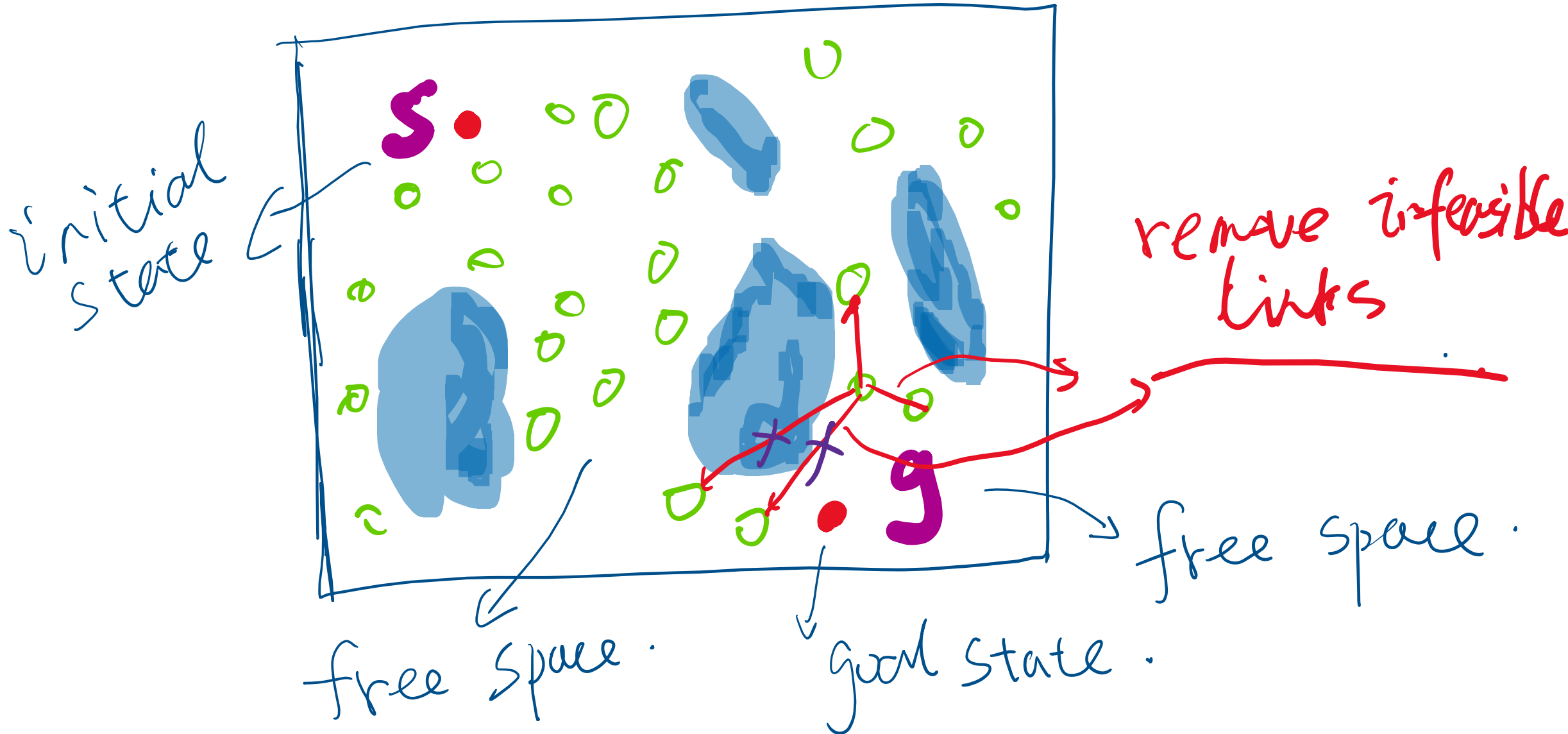


# Sampling



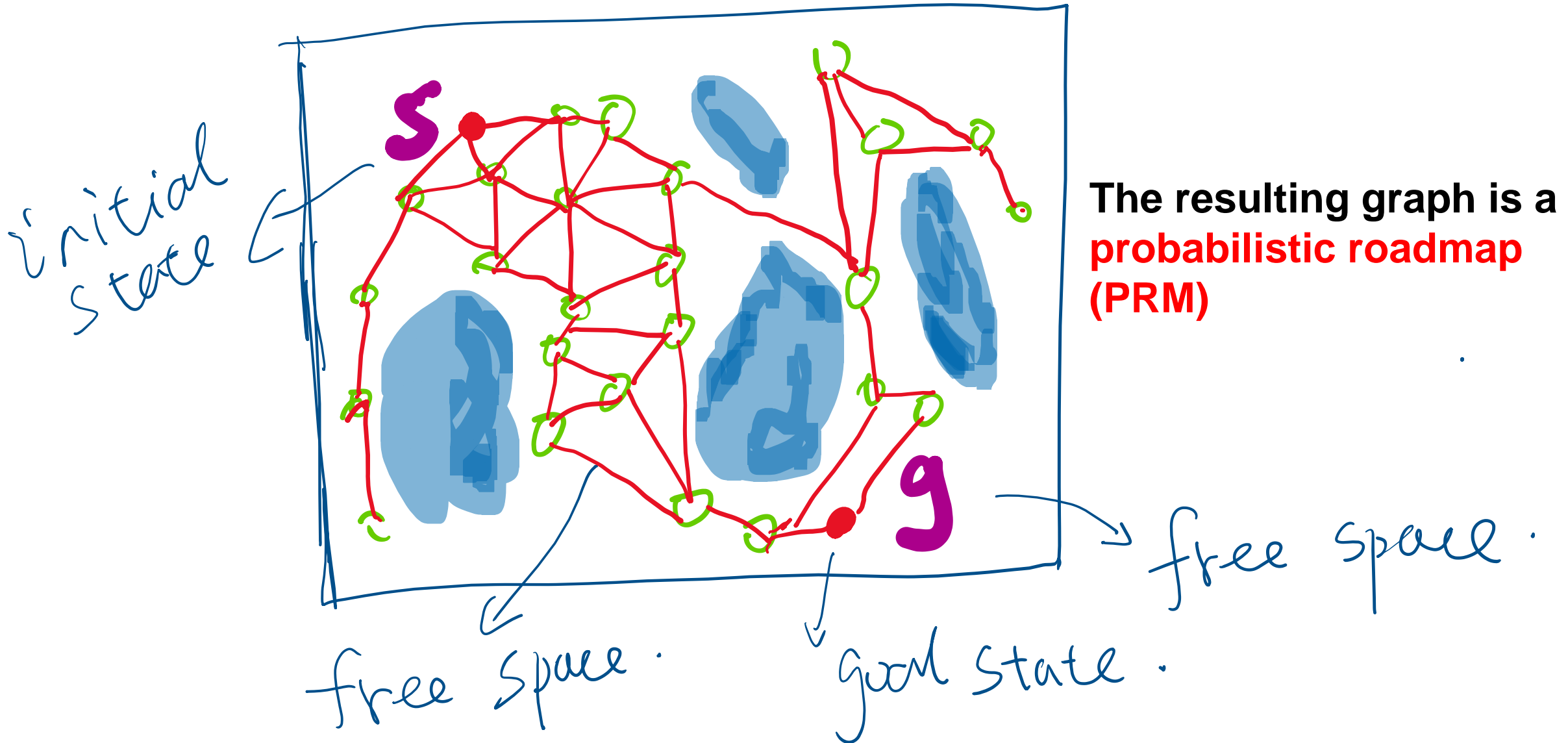


# Sampling



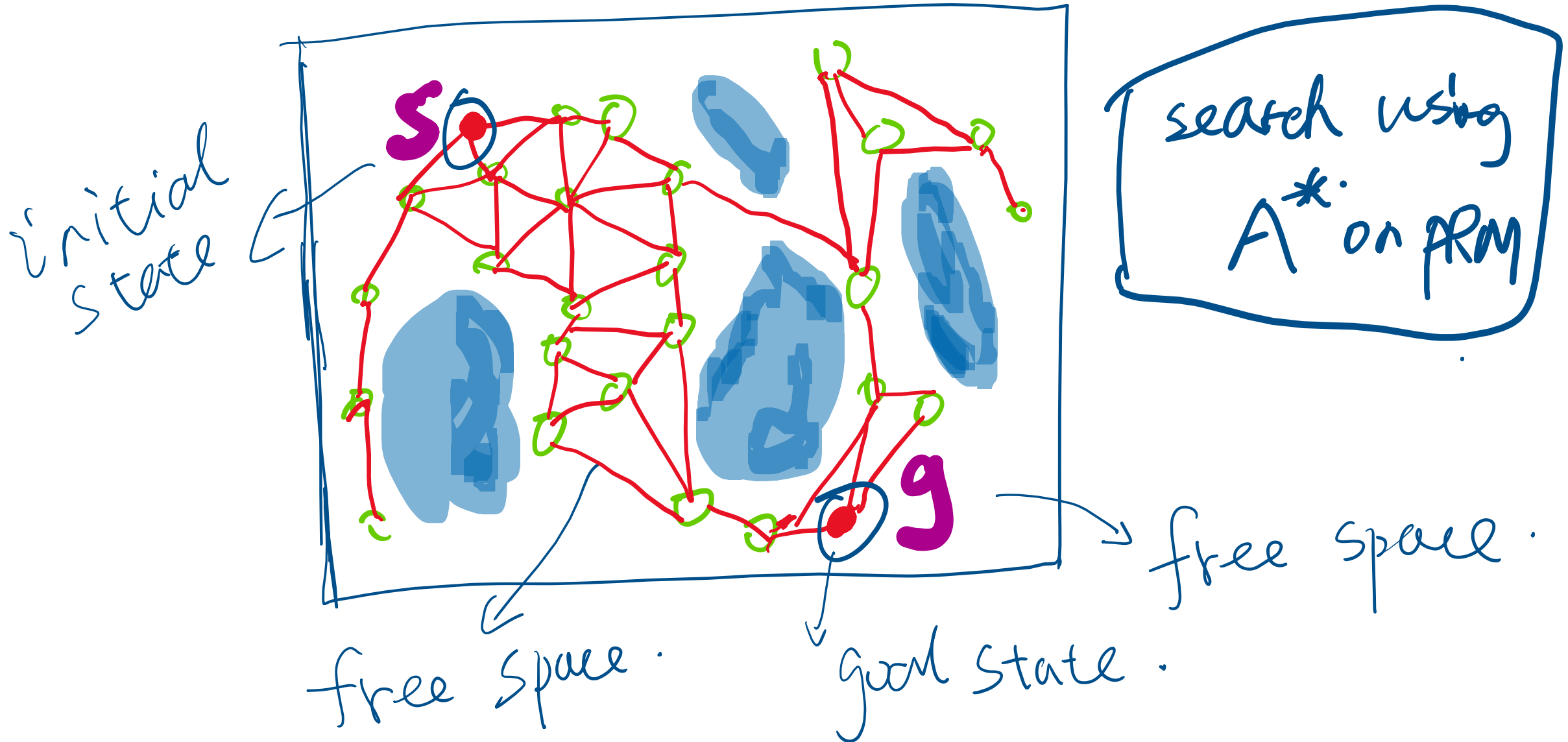


# PRM (probabilistic roadmap)



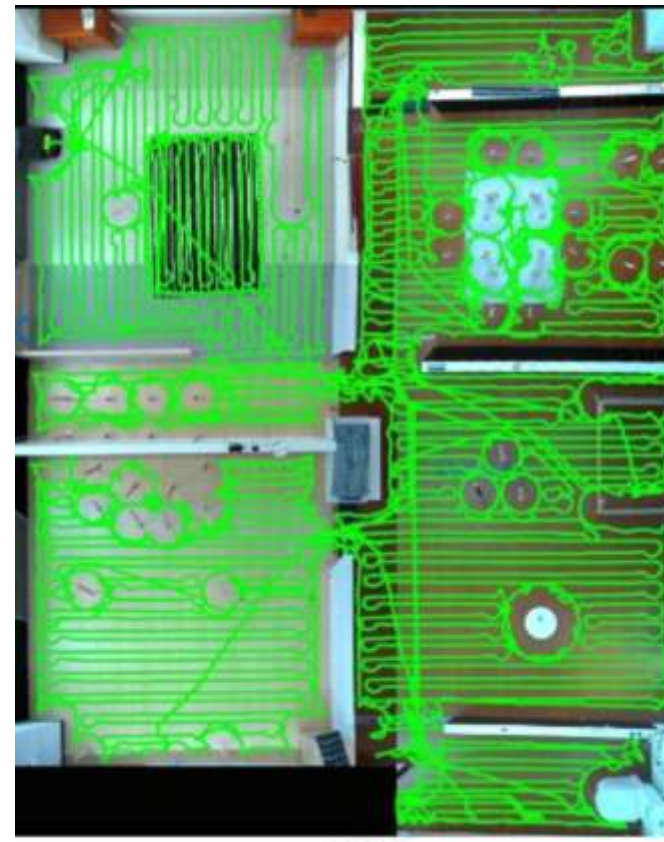
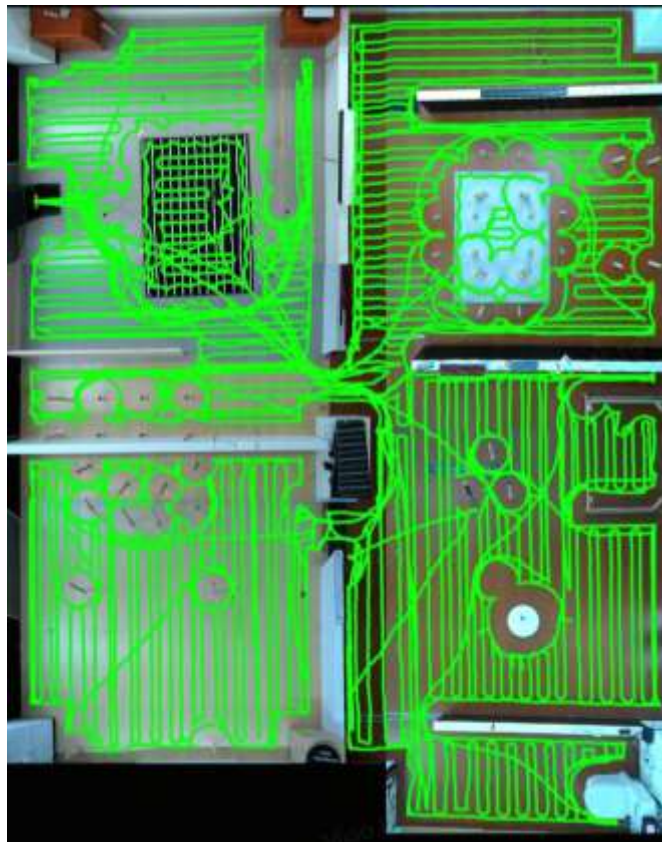
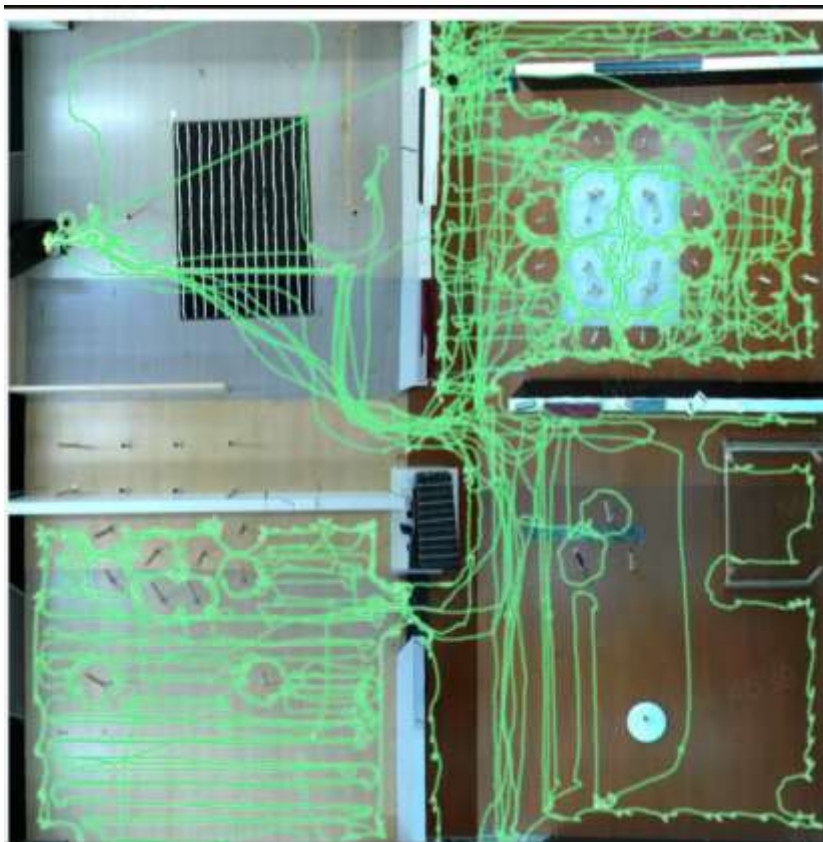


# PRM (probabilistic roadmap)





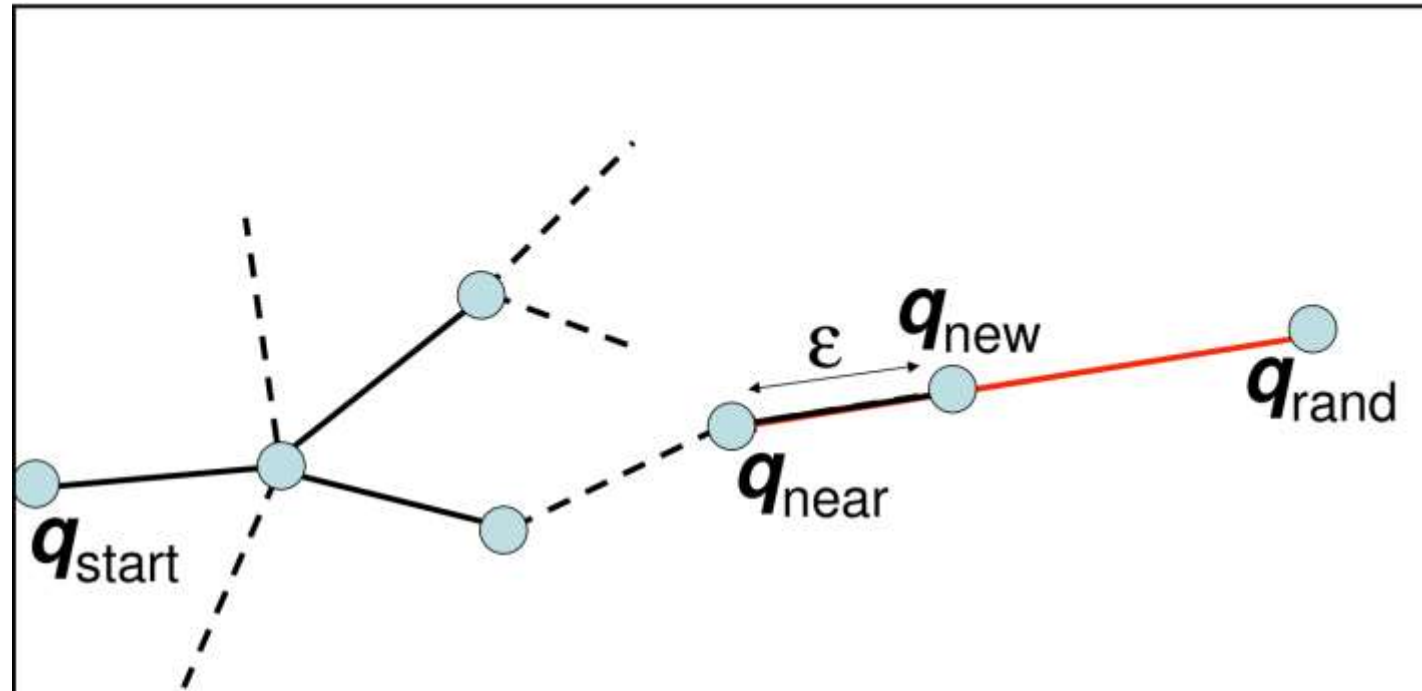
# Example





# RRT

Rapidly Exploring Random Trees



**Remarkably, we can find a solution by using *relatively few randomly* sampled points.**



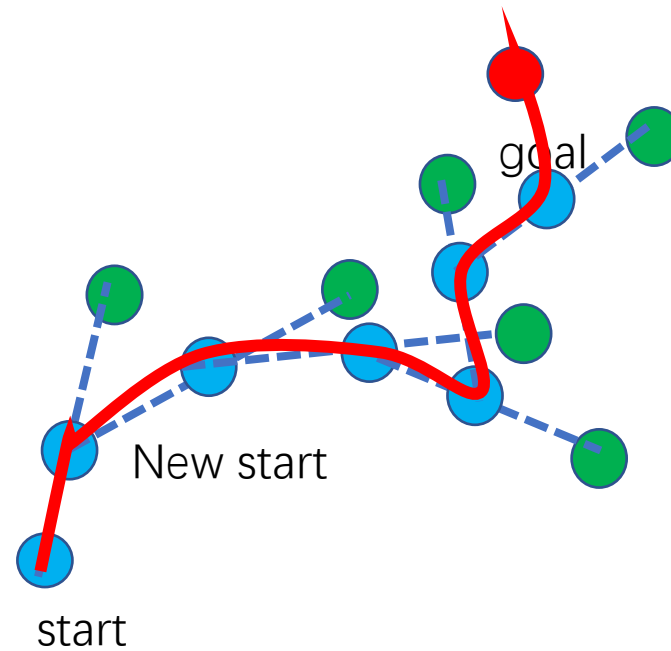
# RRT

---

## RRT Algorithm ( $x_{start}, x_{goal}, step, n$ )

---

```
1  G.initialize( $x_{start}$ )
2  for  $i = 1$  to  $n$  do
3       $x_{rand} = \text{Sample}()$ 
4       $x_{near} = \text{near}(x_{rand}, G)$ 
5       $x_{new} = \text{steer}(x_{rand}, x_{near}, \text{step\_size})$ 
6      G.add_node( $x_{new}$ )
7      G.add_edge( $x_{new}, x_{near}$ )
8      if  $x_{new} = x_{goal}$ 
9          success()
```

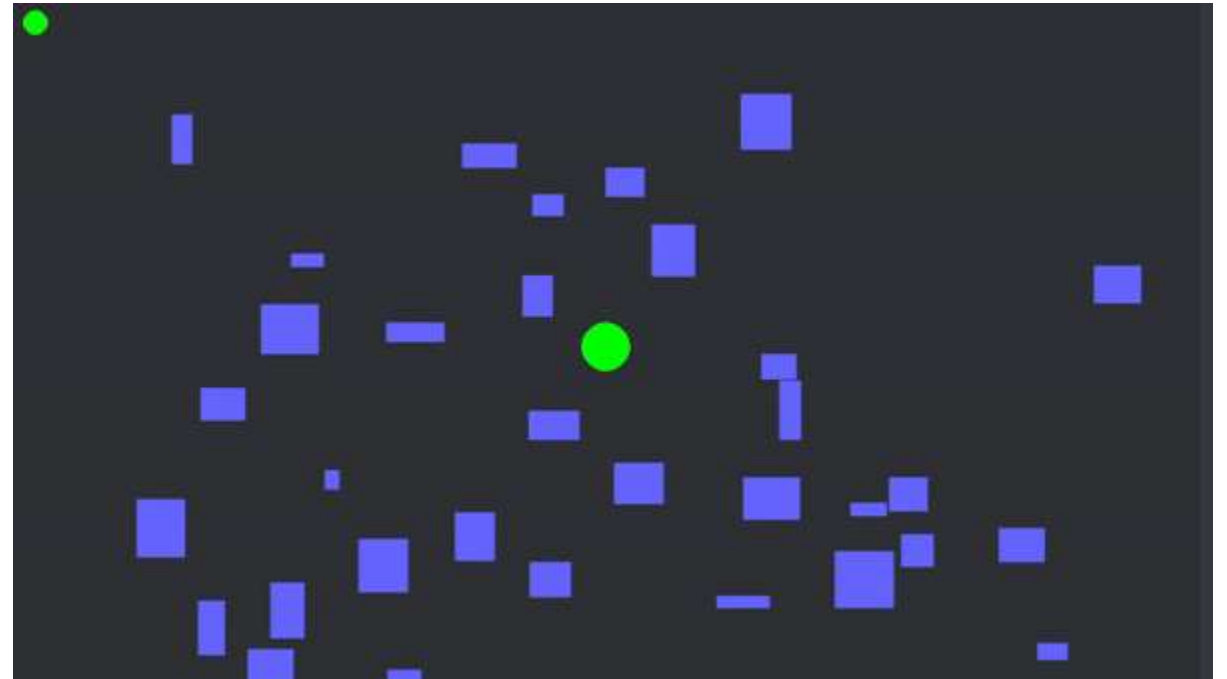
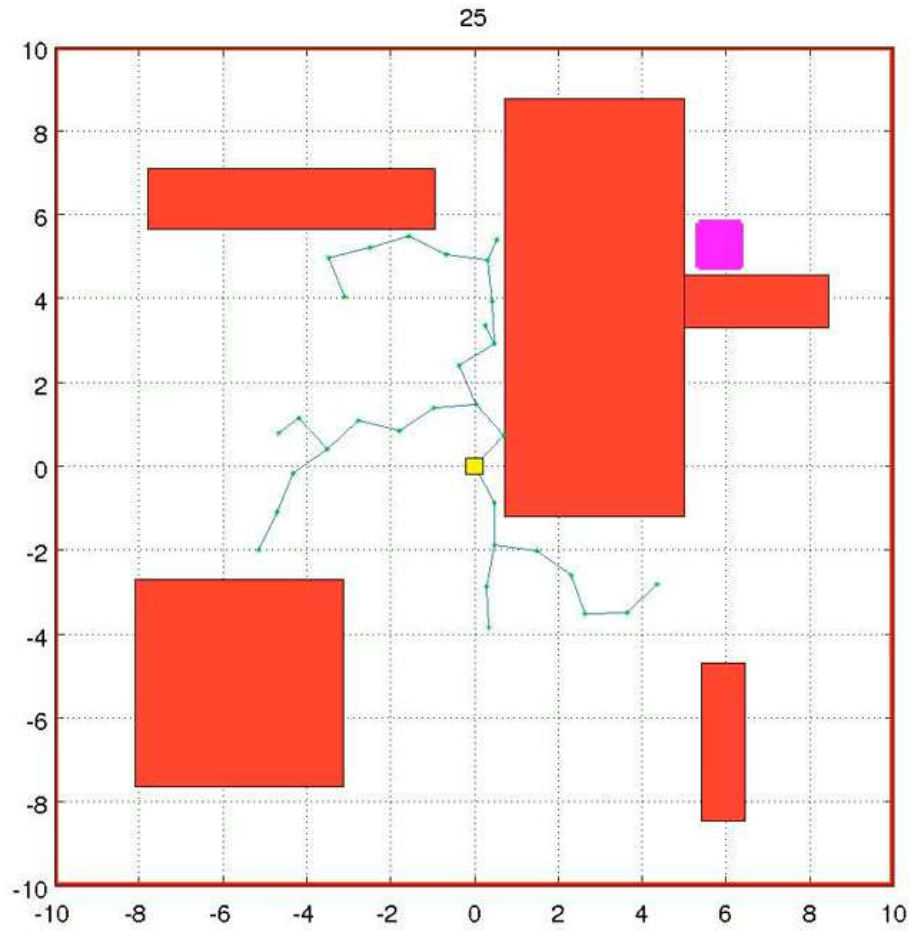


- J-C. Latombe. Robot Motion Planning. Kluwer. 1991.
- S. Lavelle. Planning Algorithms. 2006.  
<http://msl.cs.uiuc.edu/planning/>
- H. Choset et al., Principles of Robot Motion: Theory, Algorithms, and Implementations. 2006.



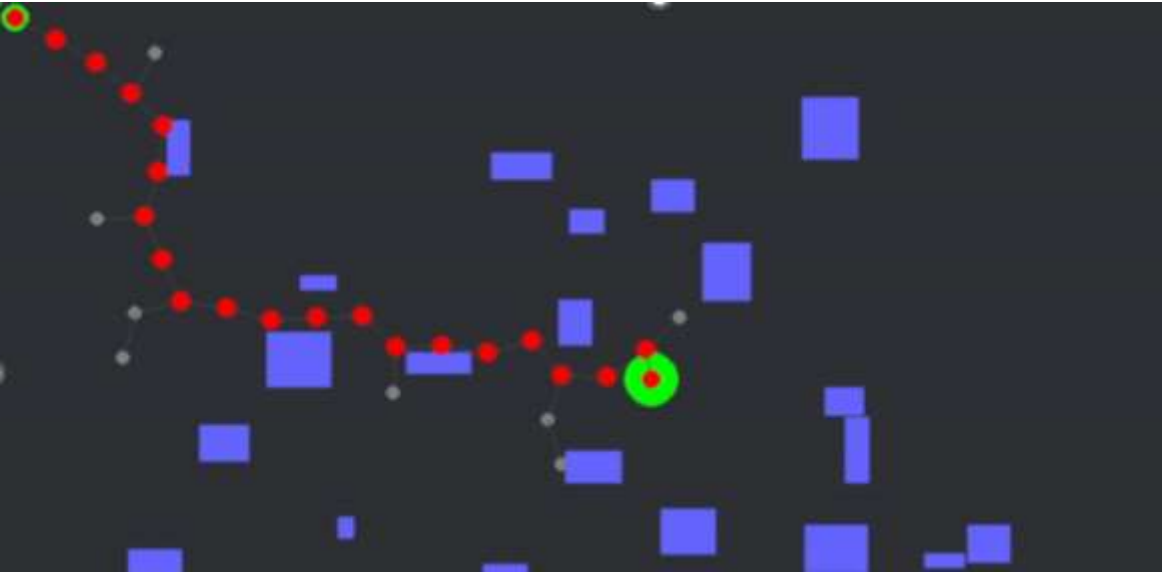


# RRT





# RRT revisit

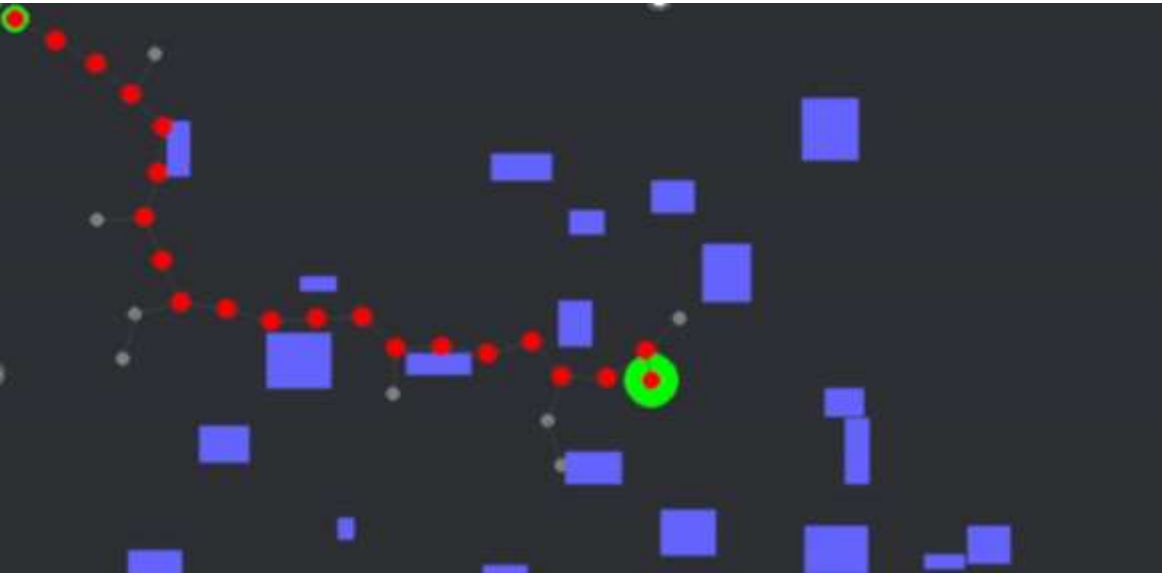


- Few control params of the solution
- Near to collisions
- Ignore trivial solution
- Path quality can be bad
- Quite different with different seeds
- Additional steps for collision checking

**What is the problem with this approach?**



# RRT revisit



**RRT is not optimal**

**What is the problem with this approach?**



# Today's Agenda

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- Reinforcement learning (~10)

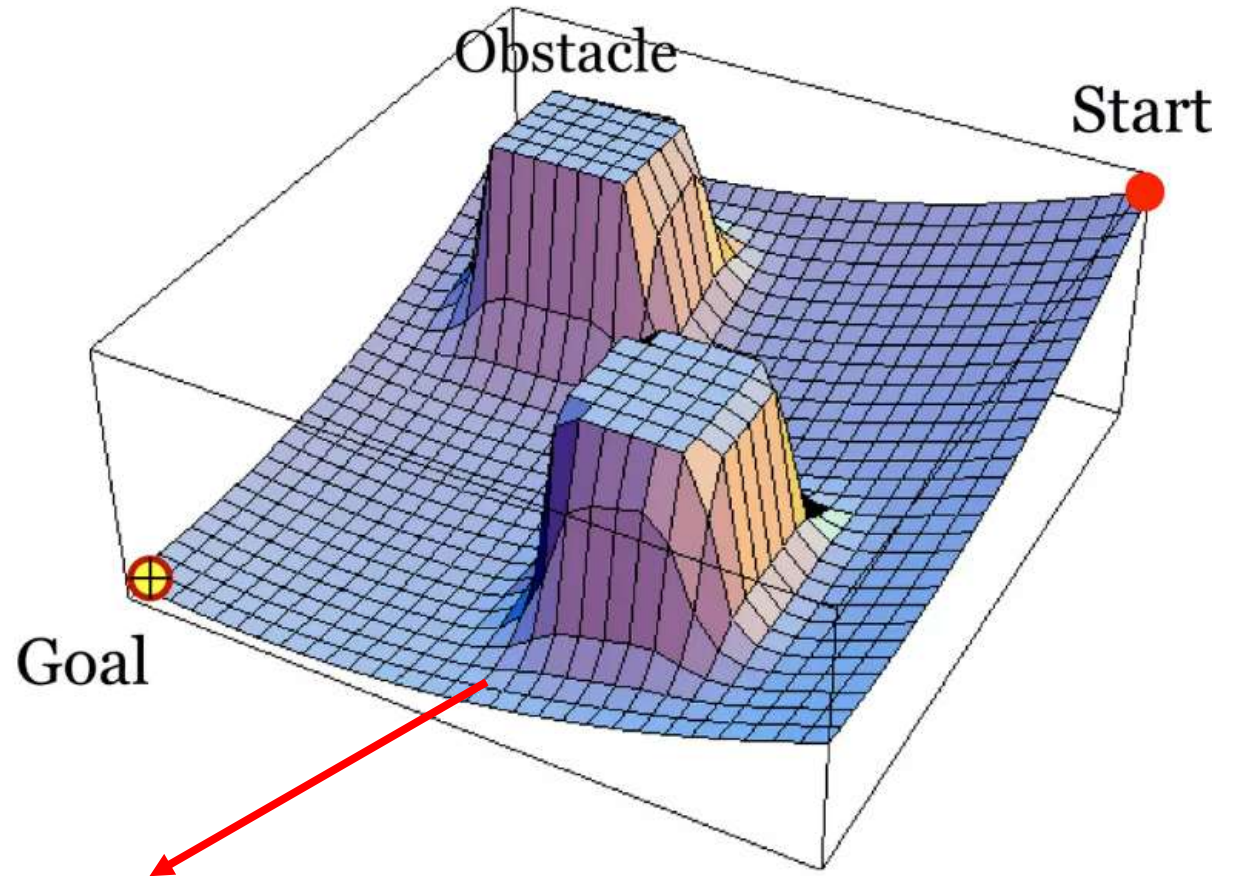


Recap of optimization-based approach

**Can we develop a motion planner  
that relies on **cost function**  
instead?**



# Potential field method

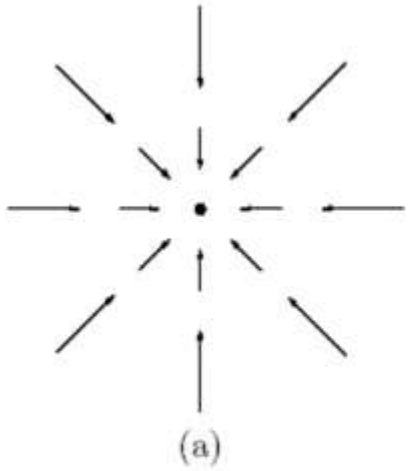


**Can we create such a cost function?**

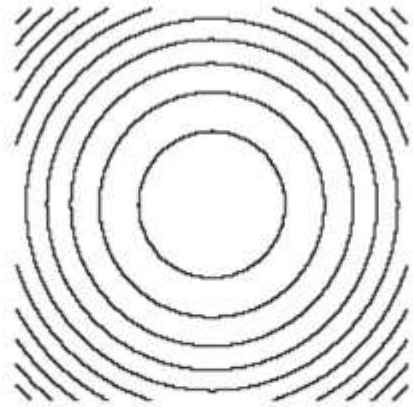


# Potential field method

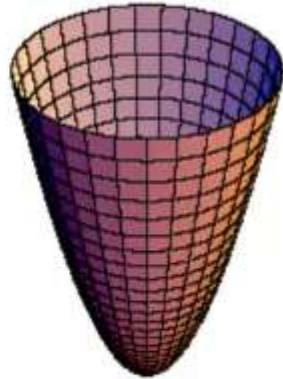
**Attraction**



(a)

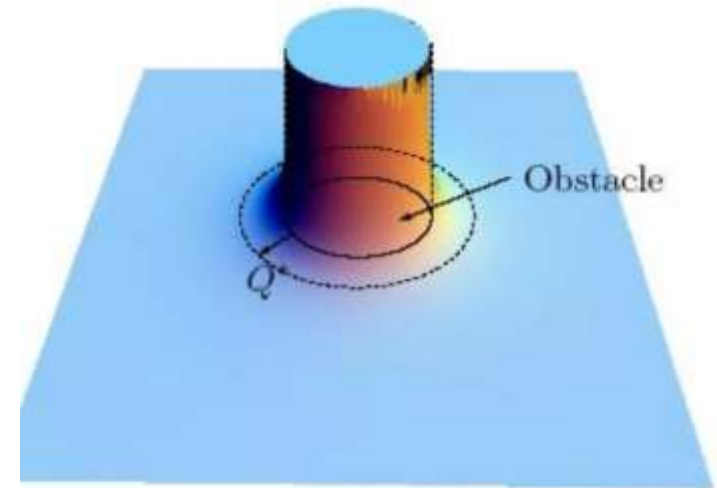


(b)



(c)

**Repulsion**

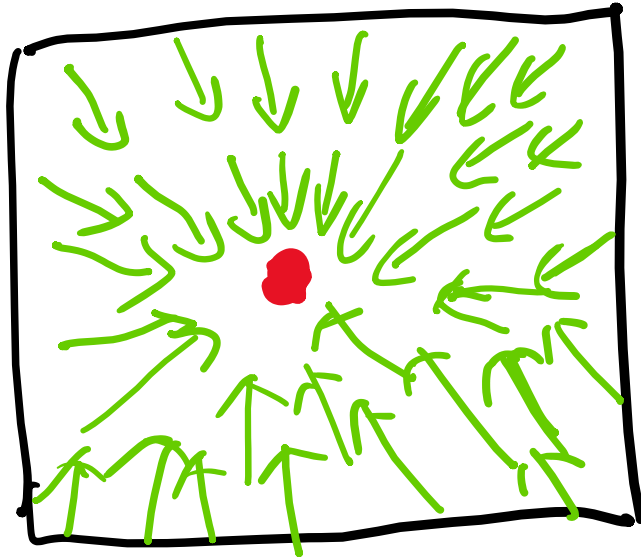


**Minimize the cost function**

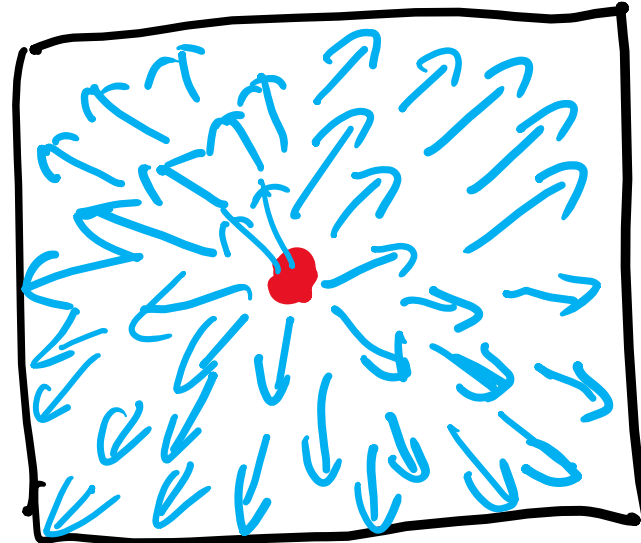


# Potential field method

**Attraction**



**Repulsion**

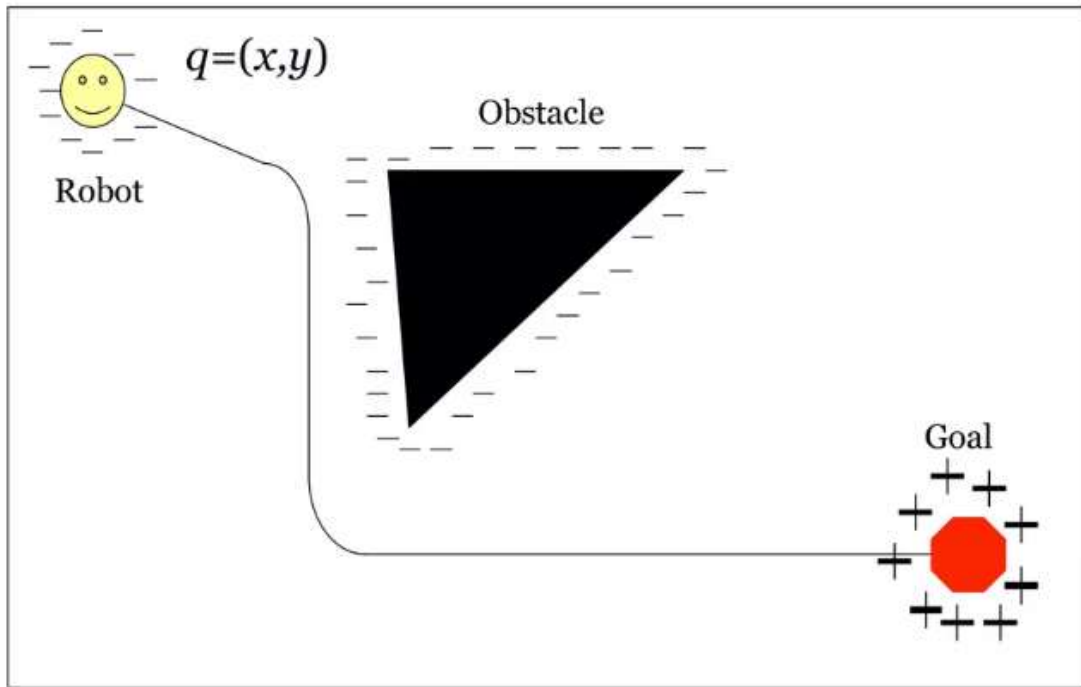


**Gradient**





# Cost function as potential



differential potential:

$$V(q)$$

artificial force:

$$F(q) = -\nabla V(q)$$

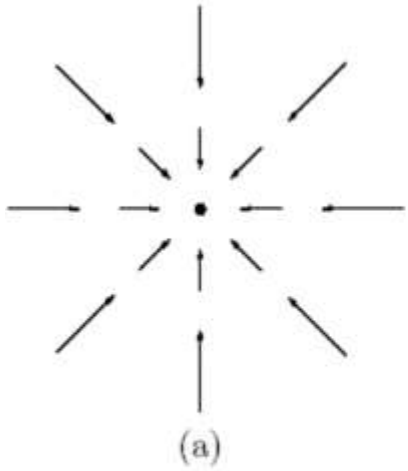
gradient

$$\nabla V(q) = \begin{bmatrix} \frac{\partial V(q)}{\partial x} \\ \frac{\partial V(q)}{\partial y} \end{bmatrix}$$

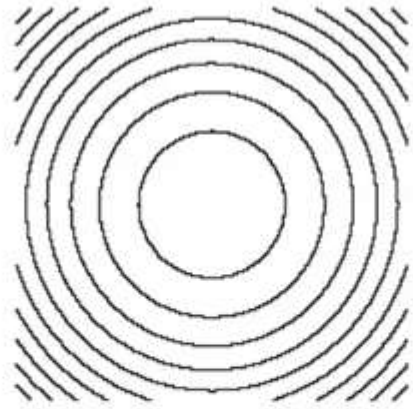


# Potential field method

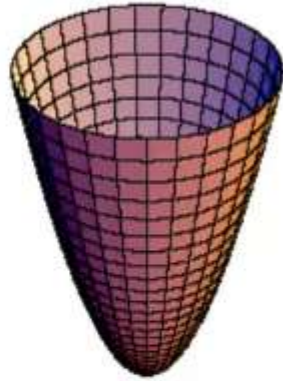
## Attraction



(a)



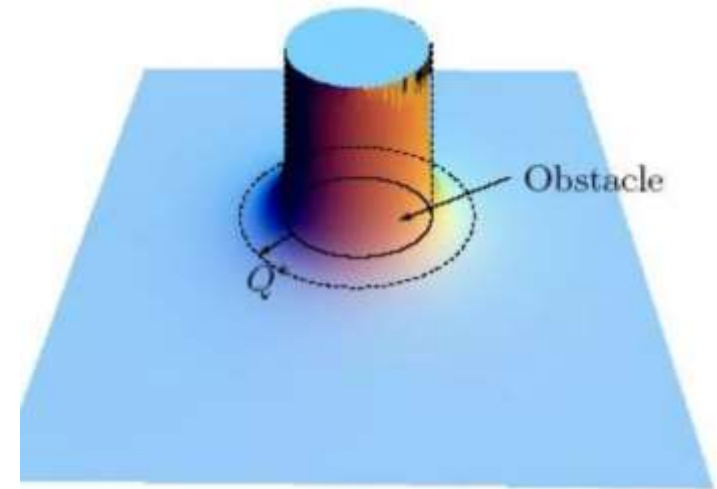
(b)



(c)

$$V_{att}(q)$$

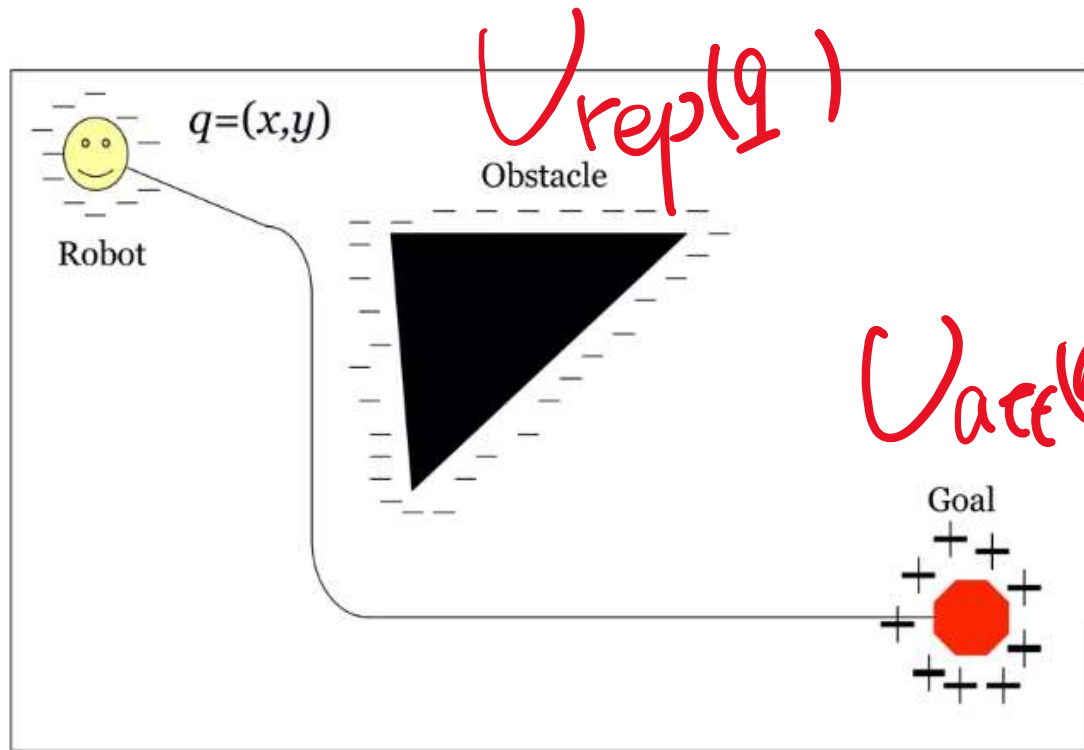
## Repulsion



$$V_{rep}(q)$$



# Potential field method



$$V(q) = V_{att}(q)$$

$$+ V_{rep}(q)$$

$V_{att}(q) \rightarrow$  move to the goal

$V_{rep}(q) \rightarrow$  avoid obstacles.



# Potential field method

attractive potential.

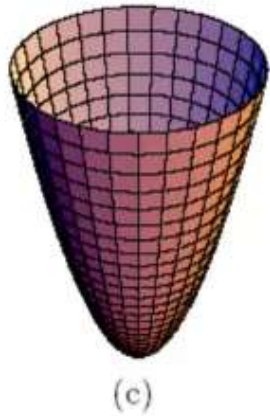
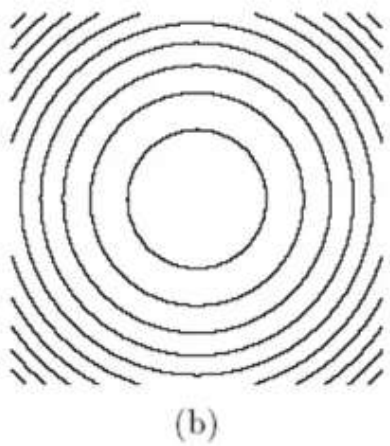
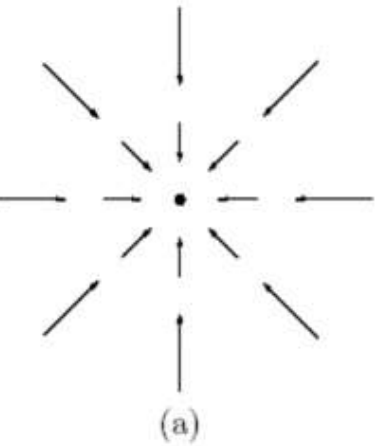
examples:

quadratic potential:

$$U_{\text{att}}(q) = \frac{1}{2} K_{\text{att}} d_{\text{goal}}^2(q)$$

$\downarrow$   
 $R^+$ , positive scaling param

$$d_{\text{goal}} = \|q - q_{\text{goal}}\|$$





# Potential field method

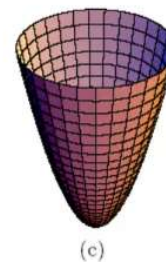
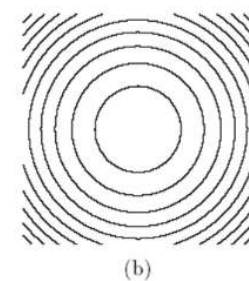
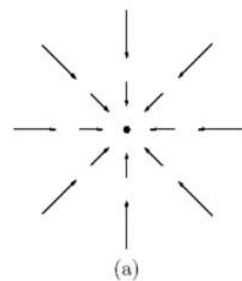
attractive potential.

★ differentiable

$$U_{att}(q) = \frac{1}{2} K_{att} d_{goal}^2(q)$$

force

$$\begin{aligned}
 \star F_{att}(q) &= - \nabla U_{att}(q) \\
 &= - K_{att} d_{goal} \nabla d_{goal} \\
 &= - K_{att} (q - q_{goal}) \\
 d_{goal} &= \|q - q_{goal}\|
 \end{aligned}$$



converge linear towards the goal



# Potential field method

repulsive potential.

key idea:

generate a force away from all known obstacles

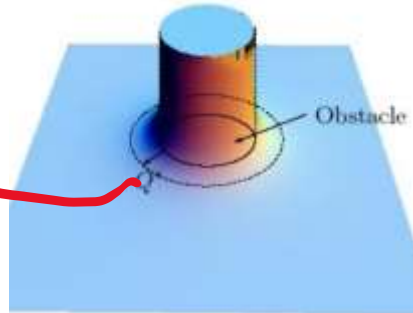
$$U_{rep}(q) = \begin{cases} \frac{1}{2} k_{rep} \cdot \left( \frac{1}{d_{obj}(q)} - \frac{1}{Q^*} \right)^2 & d_{obj}(q) \leq Q^* \\ 0 & d_{obj}(q) > Q^* \end{cases}$$

where

- ◇  $k_{rep}$  is again a scaling factor,
- ◇  $d_{obj}$  is the minimal distance from  $q$  to the object and
- ◇  $Q^*$  is the distance of influence of the object.

① Very strong: close

② Zero: far away.





# Potential field method

repulsive potential.

$$U_{rep}(q) = \begin{cases} \frac{1}{2} k_{rep} \cdot \left( \frac{1}{d_{obj}(q)} - \frac{1}{Q^*} \right)^2 & d_{obj}(q) \leq Q^* \\ 0 & d_{obj}(q) > Q^* \end{cases}$$

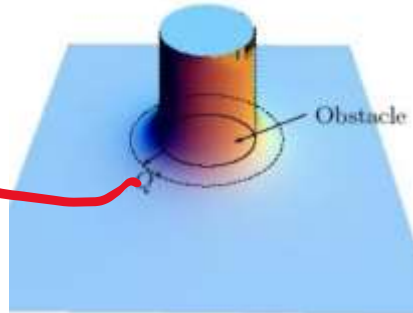
where

- ◇  $k_{rep}$  is again a scaling factor,
- ◇  $d_{obj}$  is the minimal distance from  $q$  to the object and
- ◇  $Q^*$  is the distance of influence of the object.

$d_{obj}(q) \rightarrow 0, \quad \underline{U_{rep} \rightarrow \infty}$   
?

$d_{obj} = Q^*, \quad U_{rep} \rightarrow 0$

$d_{obj} > Q^*, \quad U_{rep} = 0$





# Potential field method

repulsive force.

$$U_{rep}(q) = \begin{cases} \frac{1}{2} k_{rep} \cdot \left( \frac{1}{d_{obj}(q)} - \frac{1}{Q^*} \right)^2 & d_{obj}(q) \leq Q^* \\ 0 & d_{obj}(q) > Q^* \end{cases} \Rightarrow F_{rep}(q) = -\nabla U_{rep}(q) = \begin{cases} k_{rep} \cdot \left( \frac{1}{d_{obj}(q)} - \frac{1}{Q^*} \right) \cdot \frac{1}{d_{obj}^2} \cdot \nabla d_{obj} & d_{obj}(q) \leq Q^* \\ 0 & d_{obj}(q) > Q^* \end{cases}$$

where

- ◇  $k_{rep}$  is again a scaling factor,
- ◇  $d_{obj}$  is the minimal distance from  $q$  to the object and
- ◇  $Q^*$  is the distance of influence of the object.

$\nabla d_{obj} = \begin{bmatrix} \frac{\partial d_{obj}}{\partial x} \\ \frac{\partial d_{obj}}{\partial y} \end{bmatrix}$

$F_{rep}(q)$  when  $d_{obj} \downarrow$





# Potential field method

repulsive force.

$$U_{rep}(q) = \begin{cases} \frac{1}{2} k_{rep} \cdot \left( \frac{1}{d_{obj}(q)} - \frac{1}{Q^*} \right)^2 & d_{obj}(q) \leq Q^* \\ 0 & d_{obj}(q) > Q^* \end{cases} \Rightarrow F_{rep}(q) = -\nabla U_{rep}(q) = \begin{cases} k_{rep} \cdot \left( \frac{1}{d_{obj}(q)} - \frac{1}{Q^*} \right) \cdot \frac{1}{d_{obj}^2} \cdot \nabla d_{obj} & d_{obj}(q) \leq Q^* \\ 0 & d_{obj}(q) > Q^* \end{cases}$$

where

- ◇  $k_{rep}$  is again a scaling factor,
- ◇  $d_{obj}$  is the minimal distance from  $q$  to the object and
- ◇  $Q^*$  is the distance of influence of the object.



How to compute  $d_{obj}$ ?

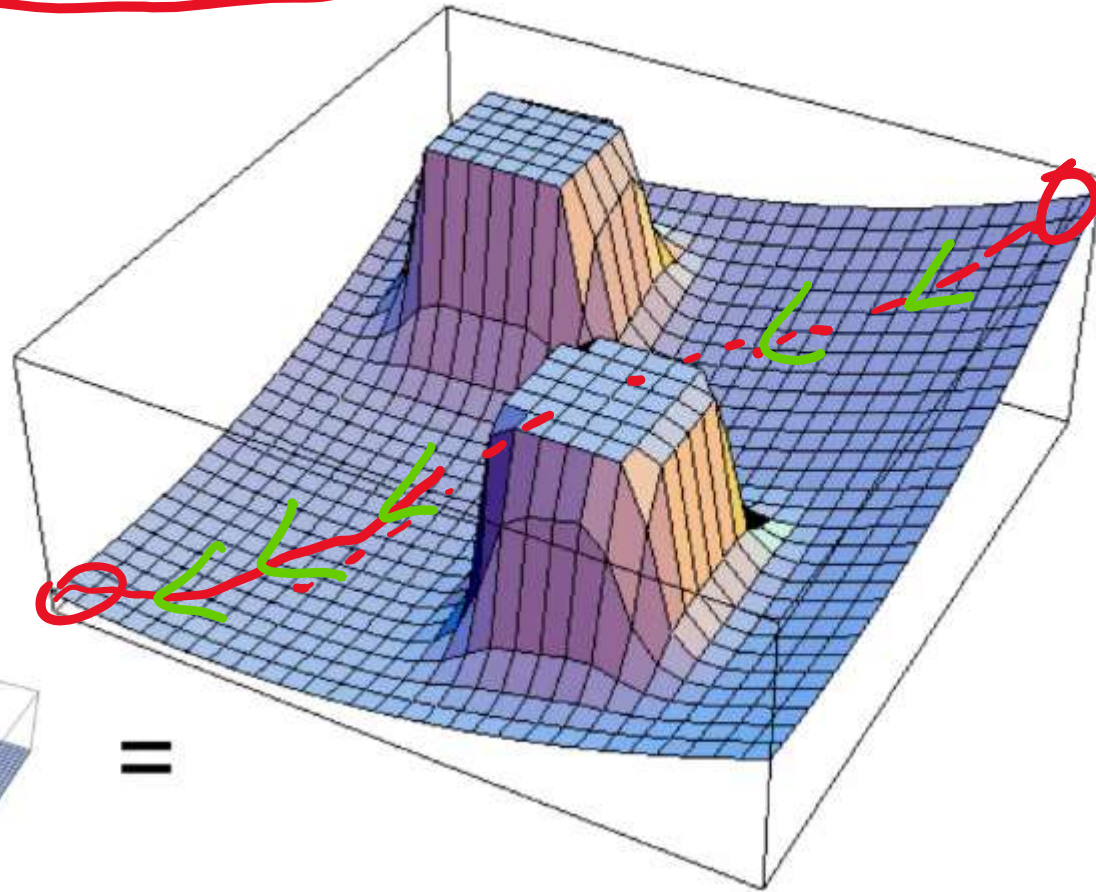
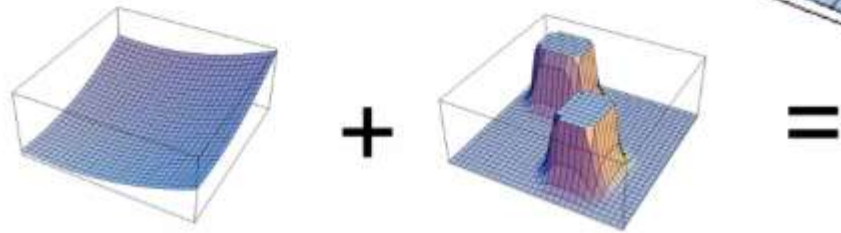
geometry



# Potential field method

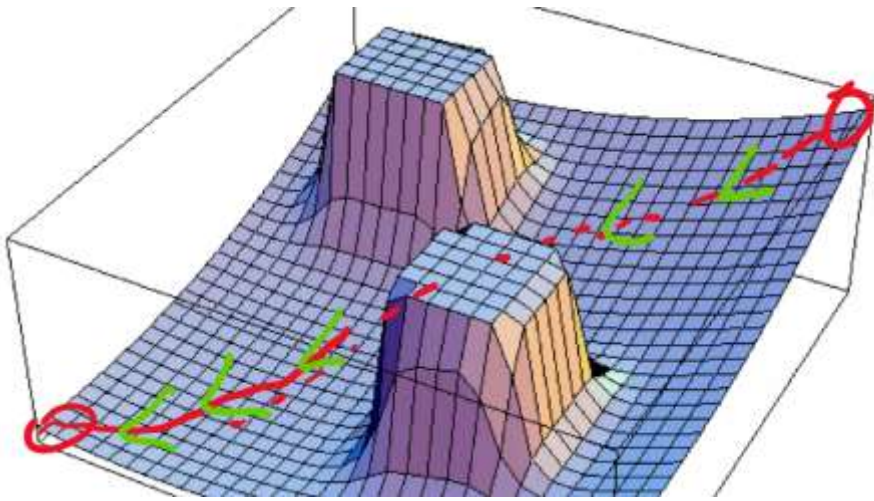
$$F(q) = F_{att}(q) + F_{rep}(q) = -\nabla U(q)$$

A first-order optimization algorithm such as **gradient descent** (also known as **steepest descent**) can be used to minimize this function by taking steps proportional to the negative of the gradient.





# Potential field method



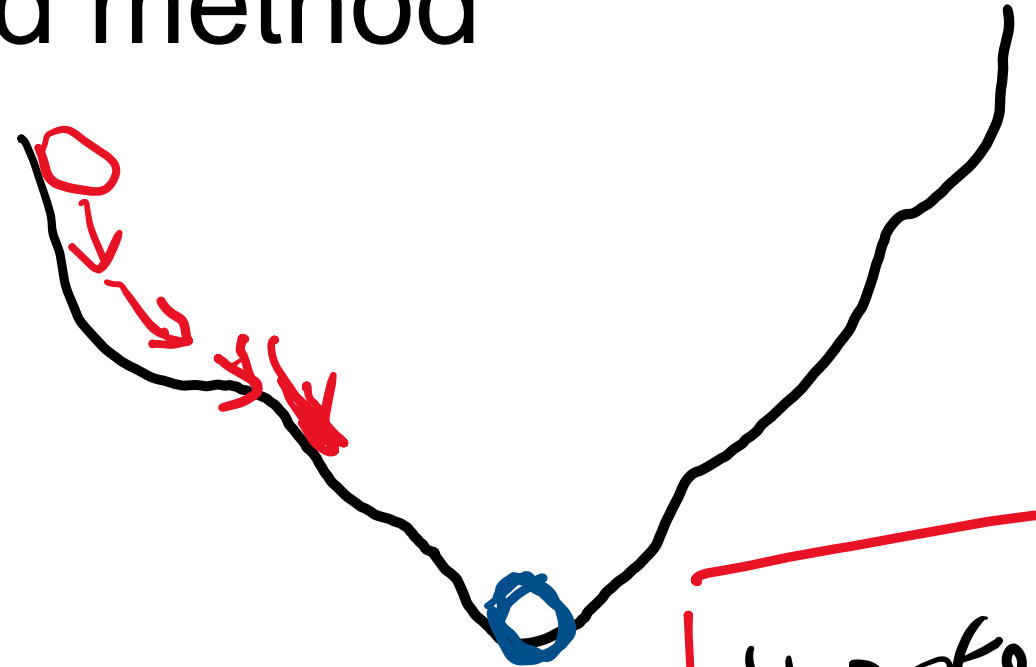
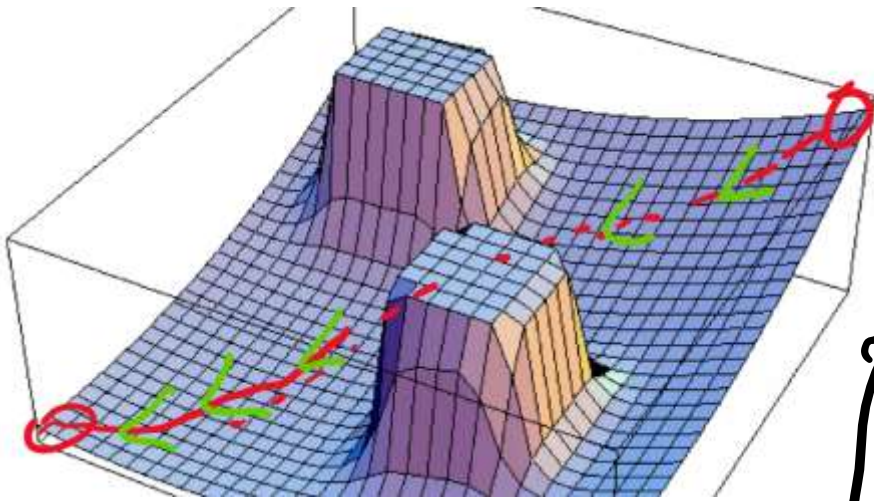
- Local minima
- Hand crafted potential function
- Hard to compute distance
- Minimal distance may not be continuous
- No passage between closely spaced obstacles
- Oscillation

problems:



# Potential field method

Reactive control:

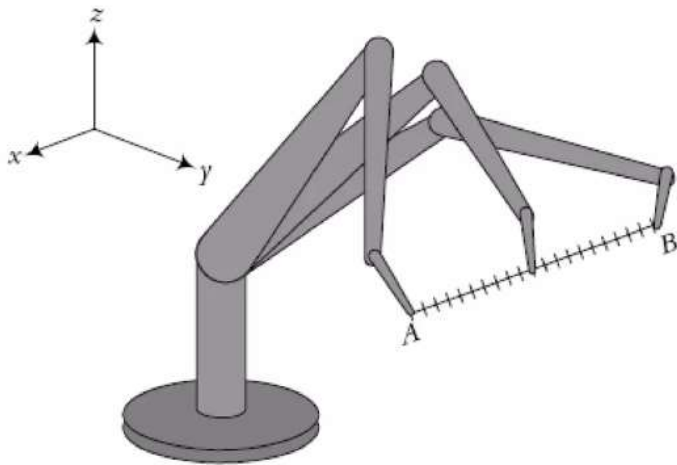


Open question:

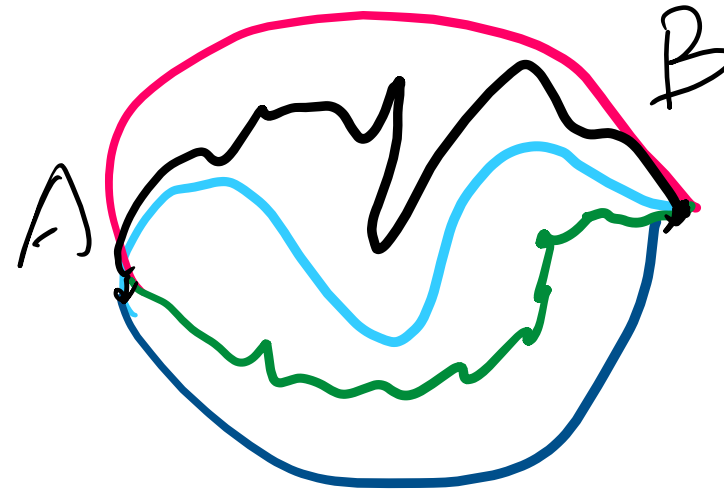
Can we design a "potential function" that can globally converge to a desired point?



# Trajectory planning (Cartesian space)



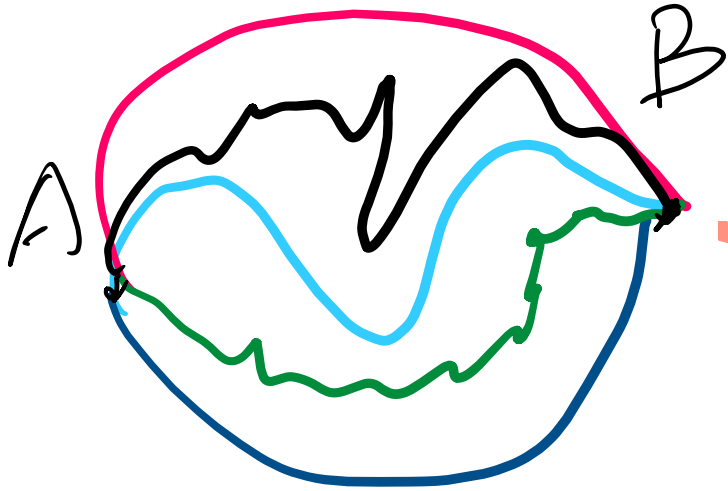
Sequential motions of a robot to follow a straight line



- **Cartesian space trajectories are very to visualize**
- **Computationally expensive: IK at each intermediate point**



# Trajectory planning (Cartesian space)



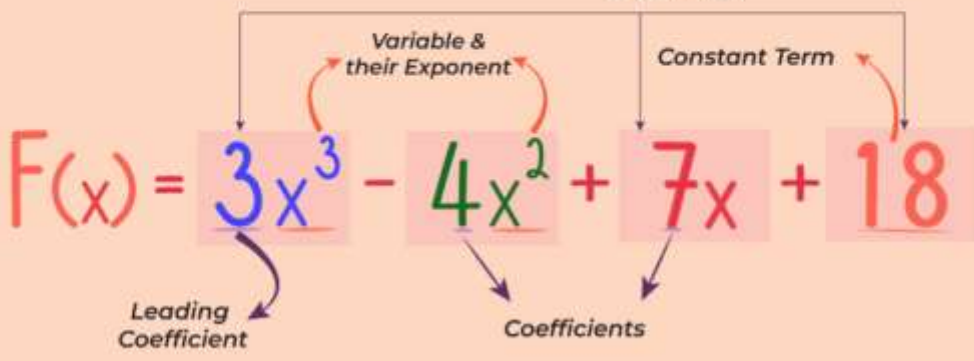
*polynomial*

any other form of trajectory? ✓

## POLYNOMIAL



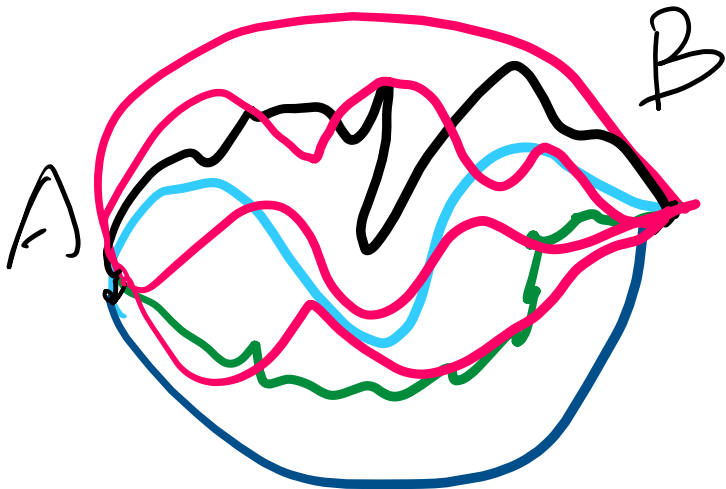
### Terms





# Trajectory planning

- The key idea of trajectory planning is to use some form of traj representation to choose the proper traj profile (polynomial...)
- This process can be applied in both joint space and Cartesian space
- Have more flexibility than sampling-base methods

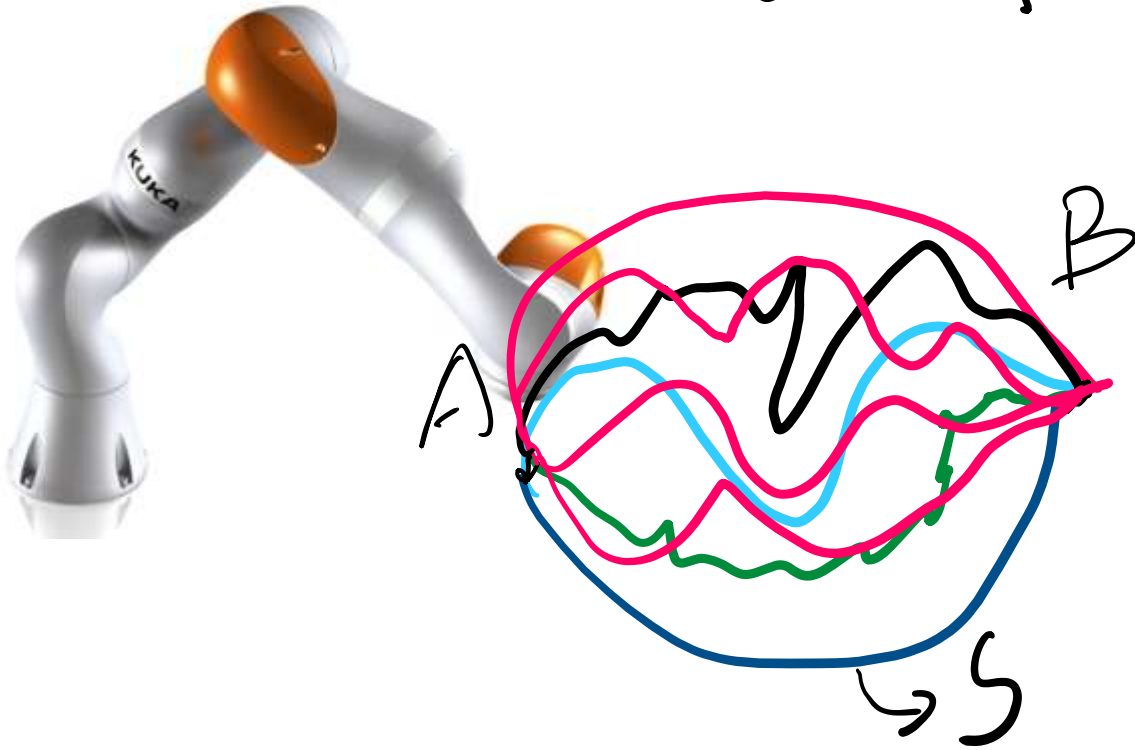


special form:



# Trajectory optimization

Cost function:  $U: S \rightarrow \mathbb{R}^+$



- path length
- efficiency
- obstacle avoidance
- uncertainty reduction
- predictability
- legibility/ intent expression
- human comfort
- naturalness

difficult to represent.





# Trajectory optimization

Cost function:  $V: S \rightarrow \mathbb{R}^+$

trj optimization:

$$S^* = \arg \min_{S \in E} V[S]$$

$$\text{s.t. } \left. \begin{array}{l} S(0) = q_s \\ S(T) = q_g \\ \text{other constraints} \end{array} \right\}$$

- path length
- efficiency
- obstacle avoidance
- uncertainty reduction
- predictability
- legibility / intent expression
- human comfort
- naturalness

difficult to represent.

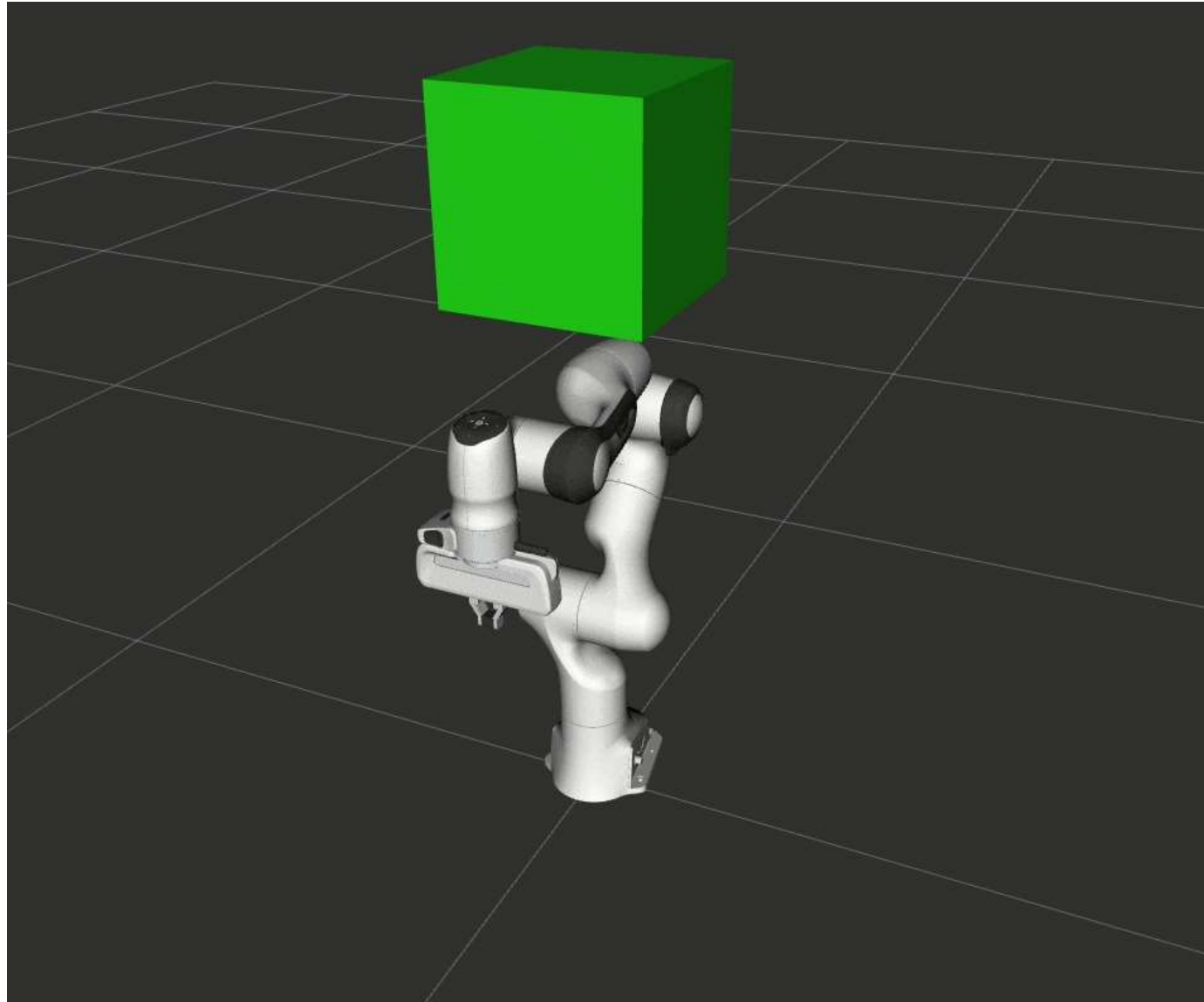


# Trajectory optimization

- **Optimization-based motion planning approaches, such as Nonlinear Programming (NLP) and Mixed-Integer Programming (MIP), solve optimization problems, and find solutions using gradient descent while satisfying constraints.**
- **For instance, CHOMP optimizes a cost functional using covariant gradient descent while TrajOpt solves a sequential convex optimization and performs convex collision checking.**
- **Various tasks including navigation, grasping, manipulation, collision-avoidance, running, cooking, and flying under various conditions.**
- **Local optimal (a general problem for nonlinear optimization)**



# Trajectory optimization





# Trajectory Optimization

$$\min_{\theta_{1:T}} \sum_t \|\theta_{t+1} - \theta_t\|^2 + \text{other costs}$$

subject to  $\theta_0 = \text{start state}$ ,  $\theta_T$  in goal set

joint limits



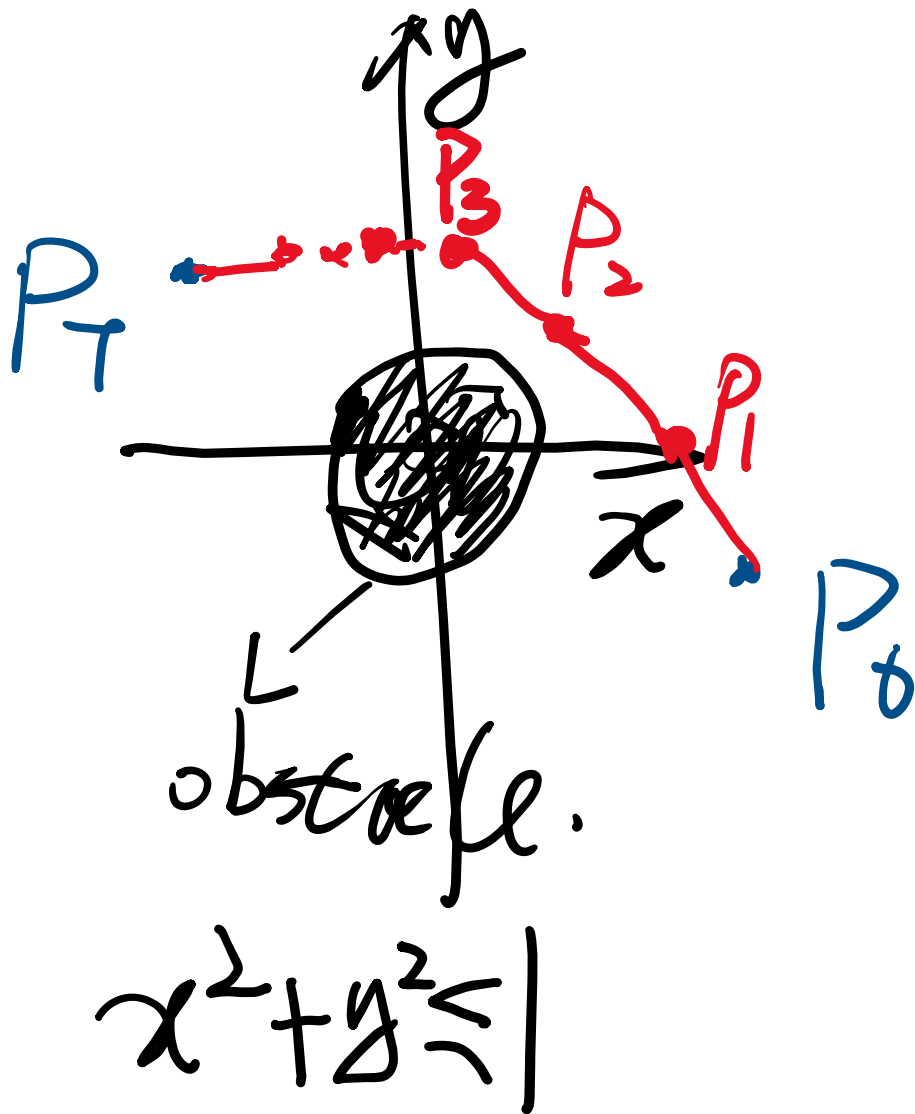
for all robot parts, for all obstacles:

no collision  $\longrightarrow$  **non-convex**

***Solution method: sequential convex optimization***



# Trajectory optimization (Example)



$$\min: \sum_n \|P_n - P_{n-1}\|^2$$

$P_0 \dots P_T$   
+ other cost

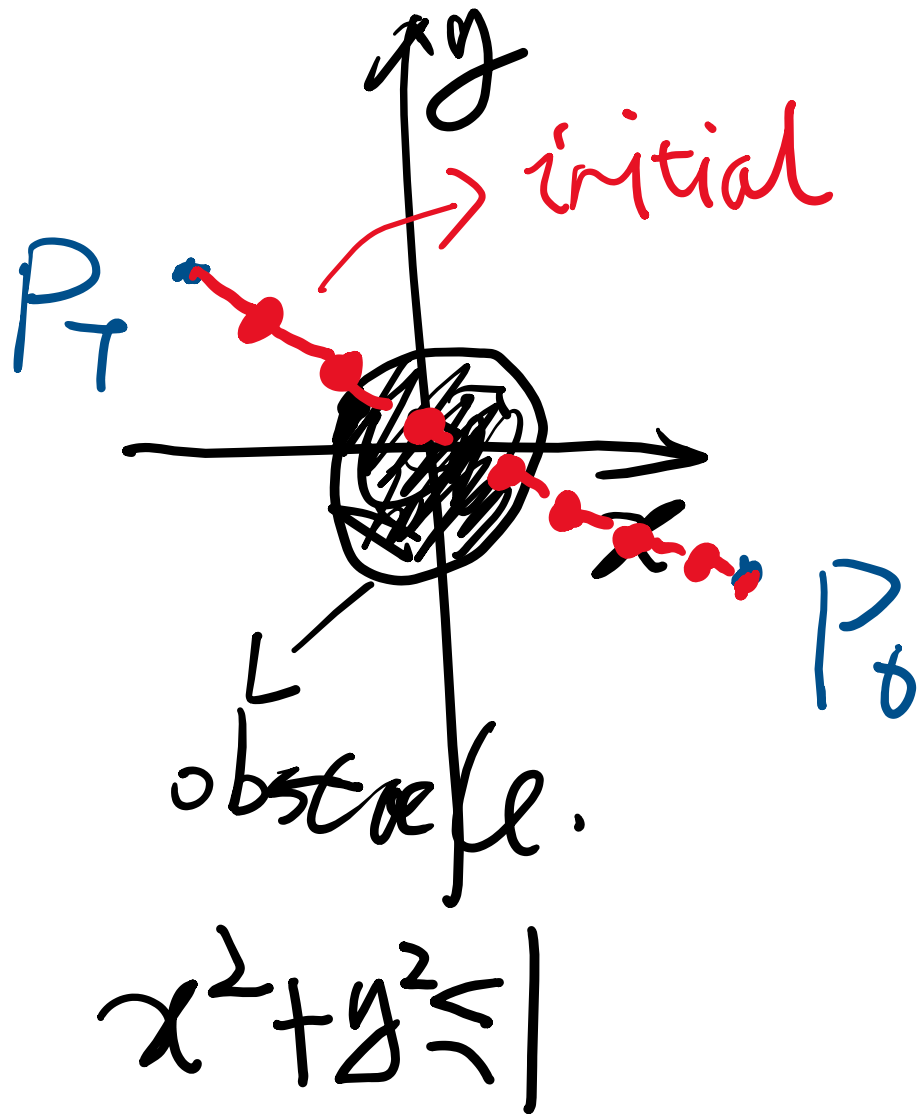
Sat.:

$$\|P_n\| \geq r + \delta_{\text{safe}}$$

try to play with this example



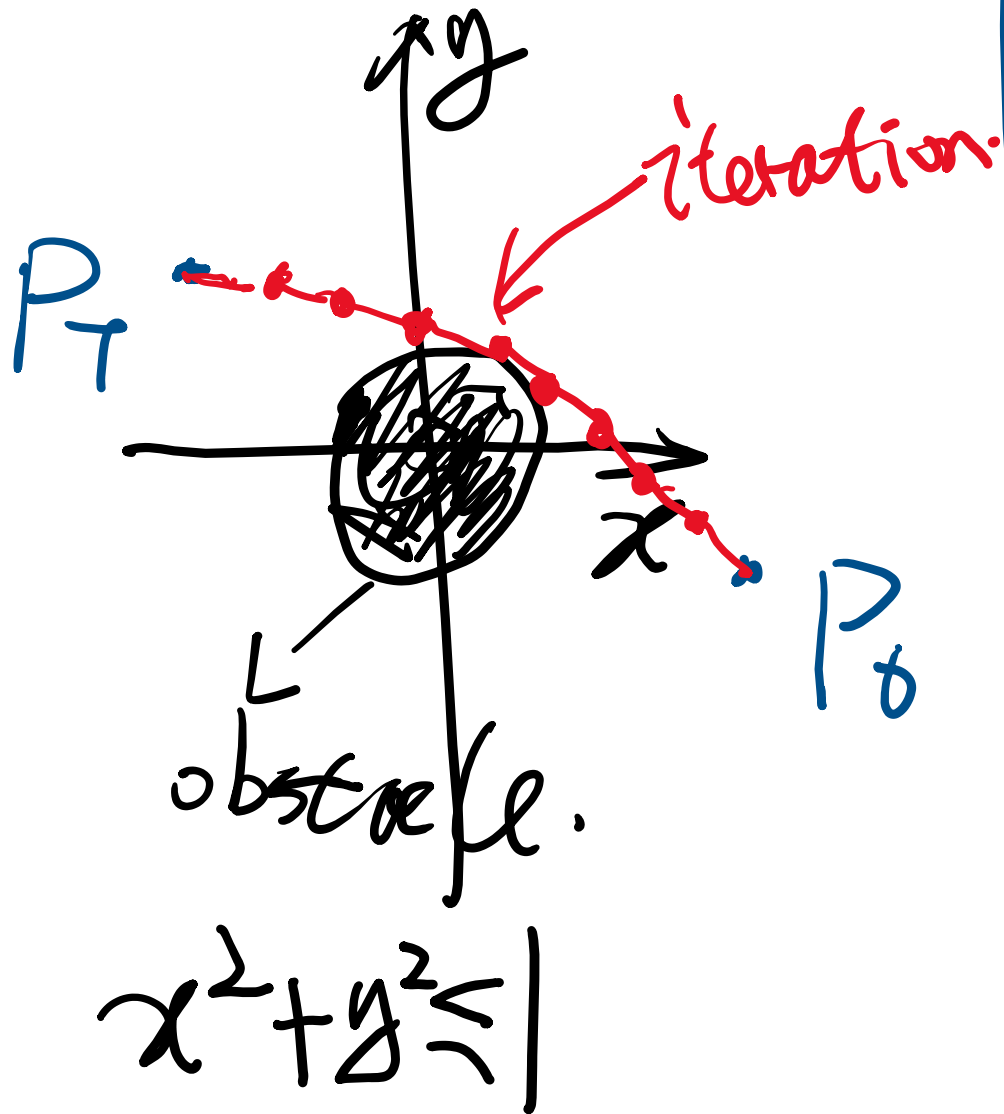
# Trajectory optimization (Example)



$$\begin{aligned} \min: & \sum_n \|P_n - P_{n-1}\|^2 \\ & P_0 \dots T \\ & + \text{other cost} \\ \text{Sat.} & \vdots \\ & \|P_n\| \geq r + \delta_{\text{safe}}. \end{aligned}$$



# Trajectory optimization (Example)



$$\min: \sum_n \|P_n - P_{n-1}\|^2$$

$P_0 \dots P_T$

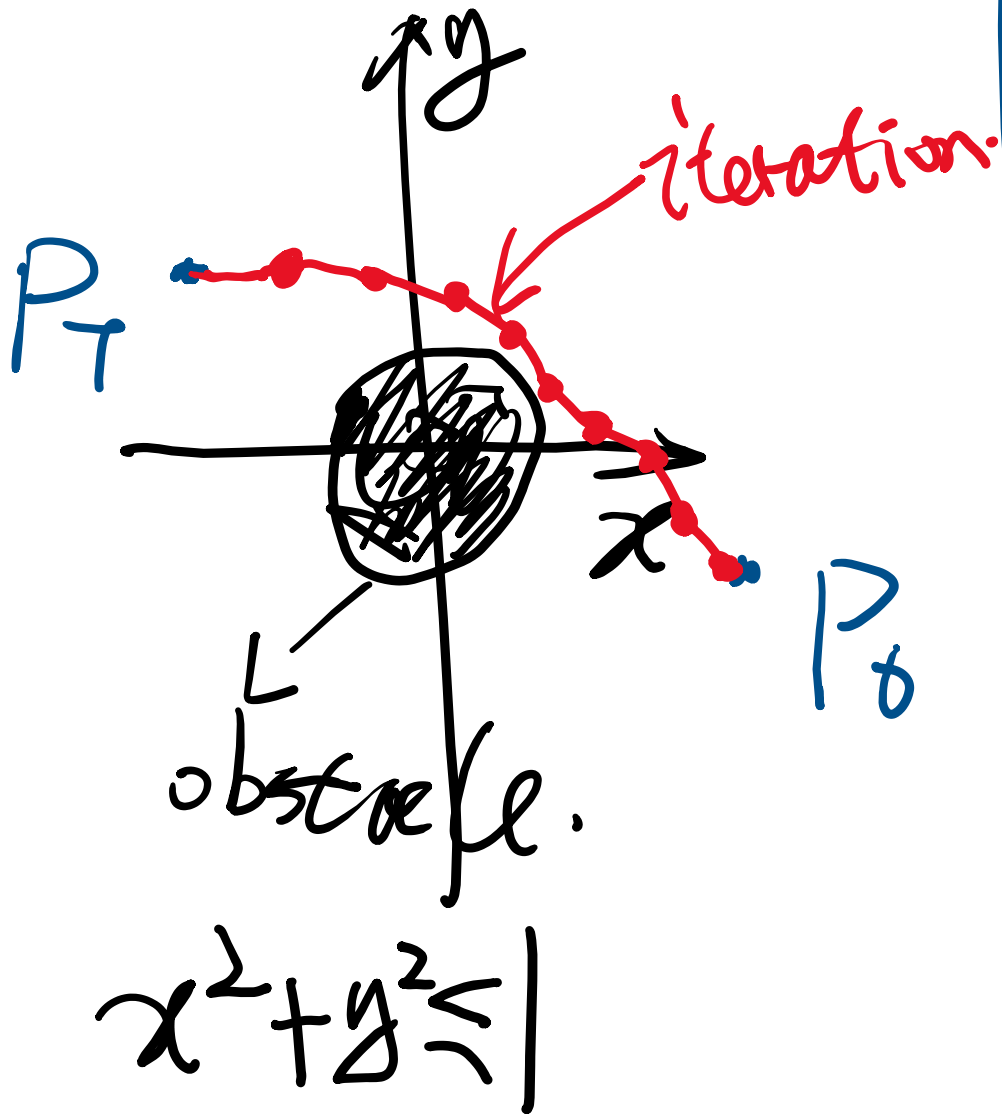
+ other cost

Sat.:

$$\|P_n\| \geq r + \delta_{\text{safe}}$$



# Trajectory optimization (Example)



$$\min: \sum_n \|P_n - P_{n-1}\|^2$$

$P_0 \dots P_T$

+ other cost

Sat.:

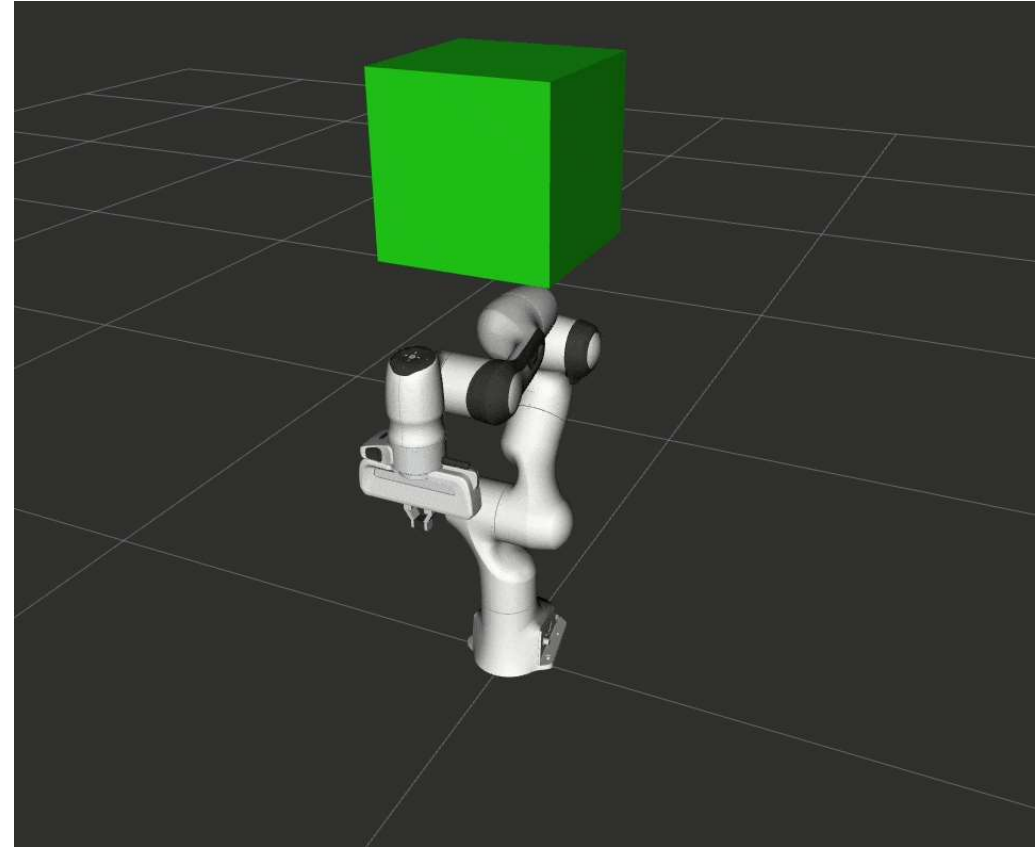
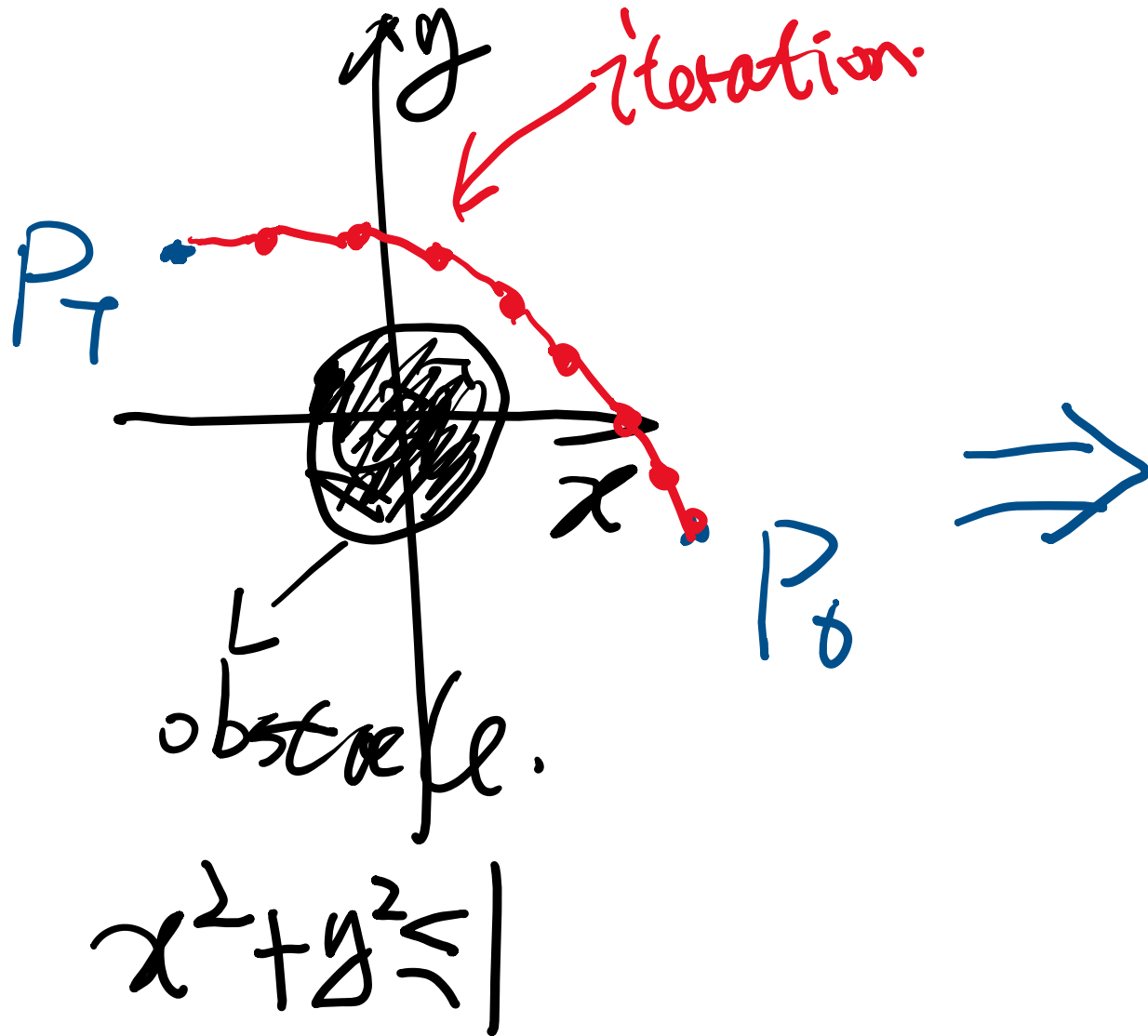
$$\|P_n\| \geq r + \delta_{\text{safe}}$$





# Trajectory optimization

(Example)



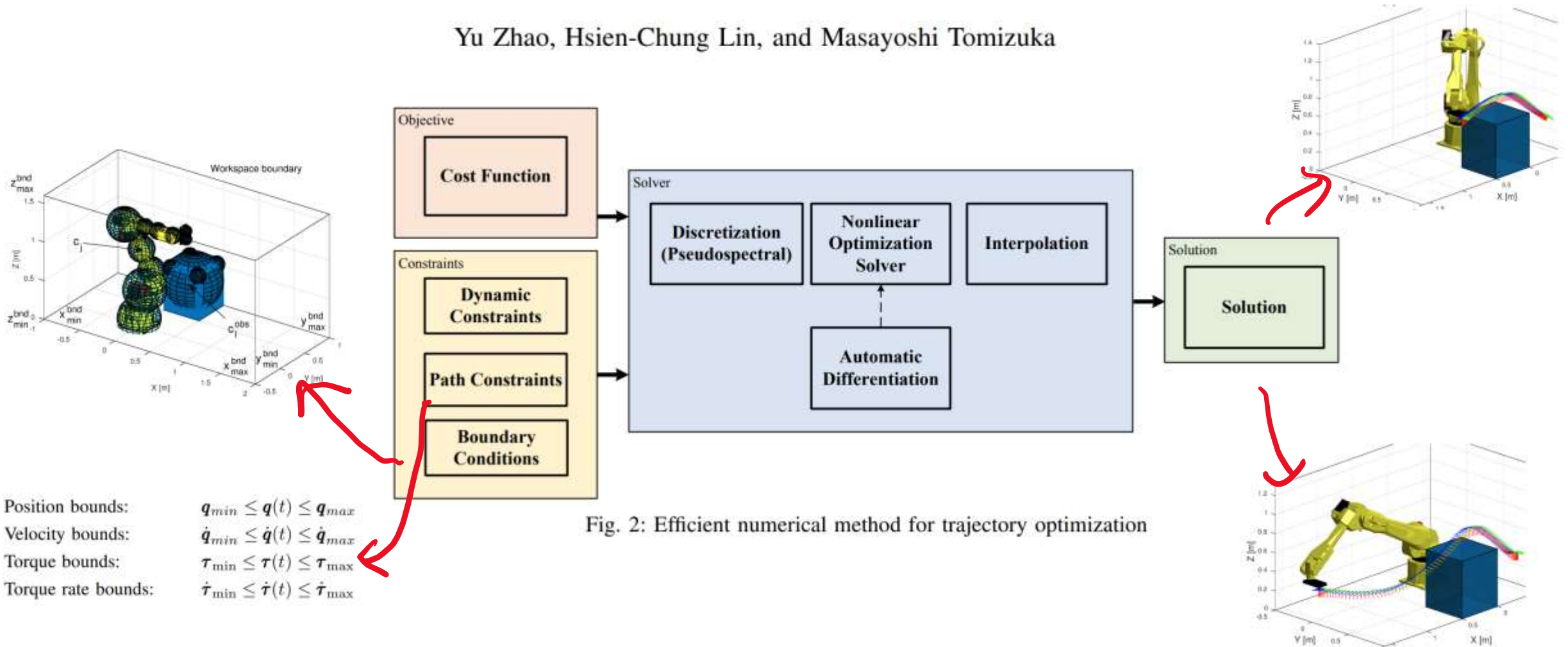
more complex constraints



# Trajectory optimization

## Efficient Trajectory Optimization for Robot Motion Planning

Yu Zhao, Hsien-Chung Lin, and Masayoshi Tomizuka



Position bounds:  $q_{\min} \leq q(t) \leq q_{\max}$   
 Velocity bounds:  $\dot{q}_{\min} \leq \dot{q}(t) \leq \dot{q}_{\max}$   
 Torque bounds:  $\tau_{\min} \leq \tau(t) \leq \tau_{\max}$   
 Torque rate bounds:  $\dot{\tau}_{\min} \leq \dot{\tau}(t) \leq \dot{\tau}_{\max}$

Fig. 2: Efficient numerical method for trajectory optimization



# Trajectory optimization

## STOMP: Stochastic Trajectory Optimization for Motion Planning

Mrinal Kalakrishnan<sup>1</sup>

Sachin Chitta<sup>2</sup>

Evangelos Theodorou<sup>1</sup>

Peter Pastor<sup>1</sup>

Stefan Schaal<sup>1</sup>

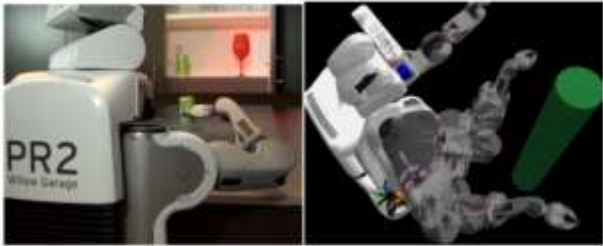
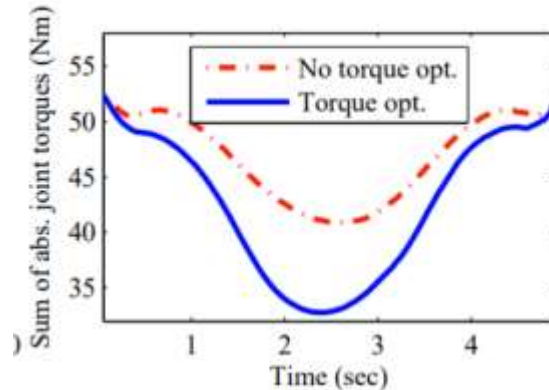
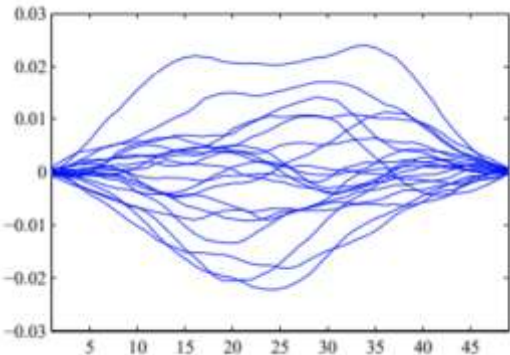


Fig. 1. (a) The Willow Garage PR2 robot manipulating objects in a household environment. (b) Simulation of the PR2 robot avoiding a pole in a torque-optimal fashion.

$$\min_{\tilde{\theta}} \mathbb{E} \left[ \sum_{i=1}^N q(\tilde{\theta}_i) + \frac{1}{2} \tilde{\theta}^T \mathbf{R} \tilde{\theta} \right]$$

STOMP is an algorithm that performs local optimization, i.e. it finds a locally optimum trajectory rather than a global one. Hence, performance will vary depending on the initial



sample locally  
optimize locally



# Today's Agenda

- Recap of sampling-based approach (~10)
- Recap of optimization-based approach (~20)
- **Drawback of sampling and optimization (~5)**
- Recap of perception-action loop (~2)
- Learning-based motion planning (~5)
- Imitation learning (~20)
- Reinforcement learning (~10)



# Drawback of sampling and optimization-based approaches

- **Flexibility**
- **Human-like**
- **Reactive**
- **Sensory feedback**



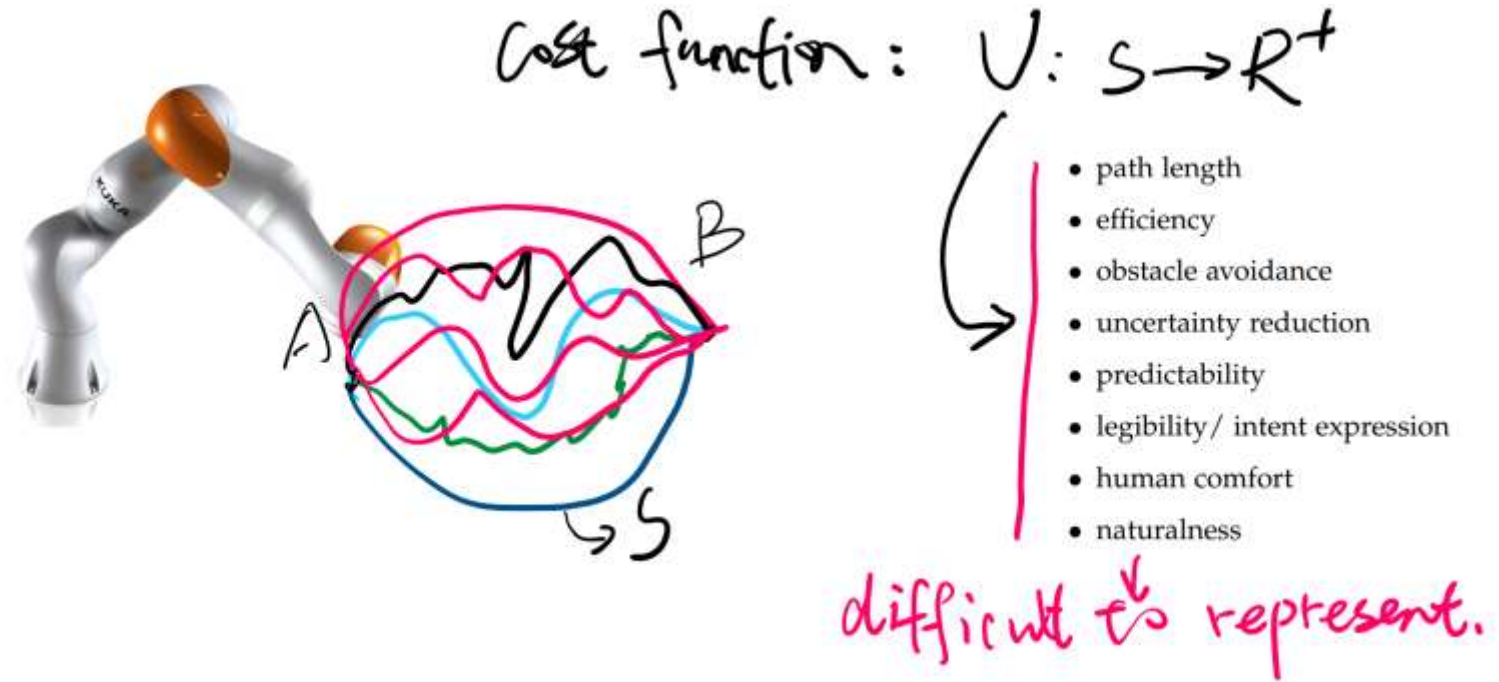
# Drawback of sampling and optimization-based approaches

- **Flexibility**

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# Drawback of sampling and optimization-based approaches

- Flexibility
- **Human-like**
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# Drawback of sampling and optimization-based approaches

- Flexibility
- **Human-like**
- Reactive
- Sensory feedback

[https://www.youtube.com/watch?v=-9JrDMBg2HE&t=38s&ab\\_channel=MITCSAIL](https://www.youtube.com/watch?v=-9JrDMBg2HE&t=38s&ab_channel=MITCSAIL)





# Drawback of sampling and optimization-based approaches

- Flexibility
- Human-like
- **Reactive**
- Sensory feed

Reactive Human-to-Robot Handovers of Arbitrary Objects





# Drawback of sampling and optimization-based approaches

- Flexibility
- Human-like
- **Reactive**
- Sensory feed

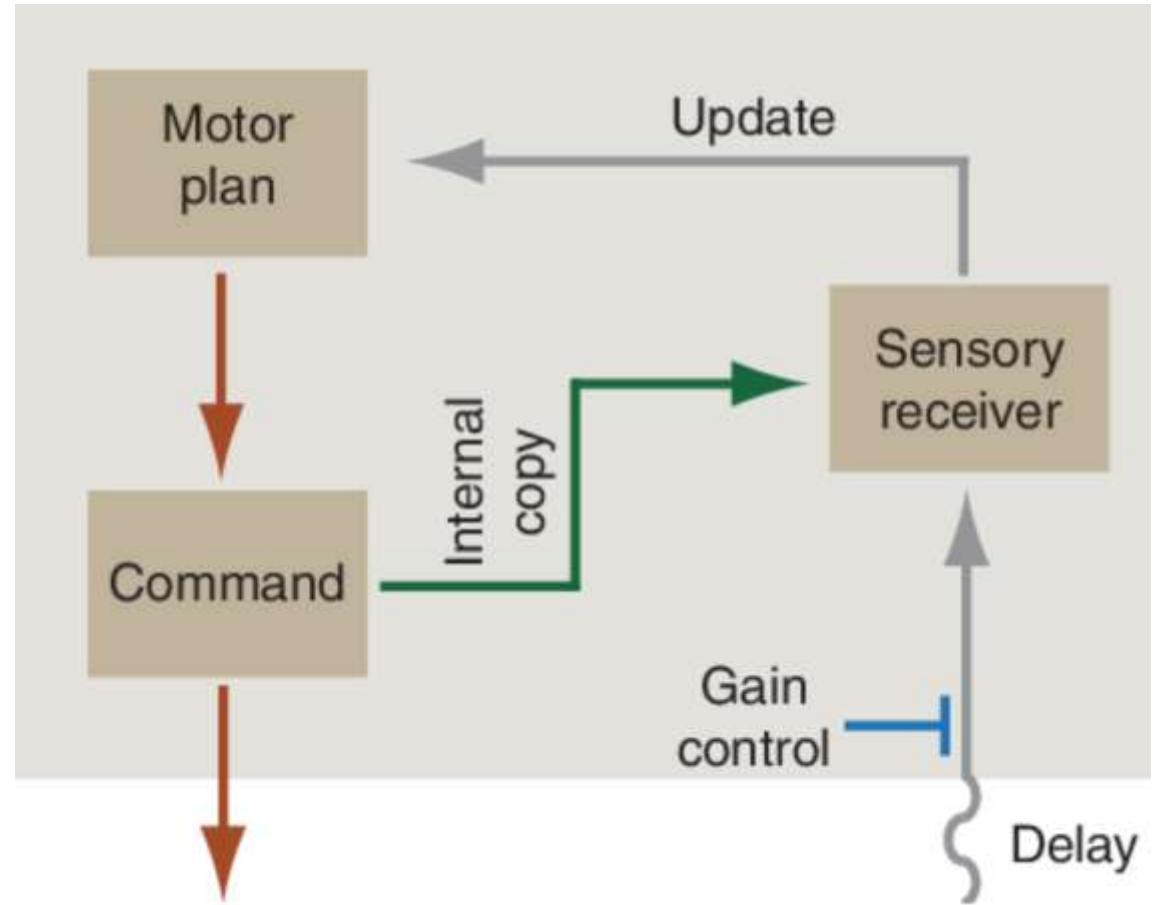


[https://research.nvidia.com/publication/2021-03\\_reactive-human-robot-handovers-arbitrary-objects](https://research.nvidia.com/publication/2021-03_reactive-human-robot-handovers-arbitrary-objects)



# Drawback of sampling and optimization-based approaches

- Flexibility
- Human-like
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- **Sensory feedback**



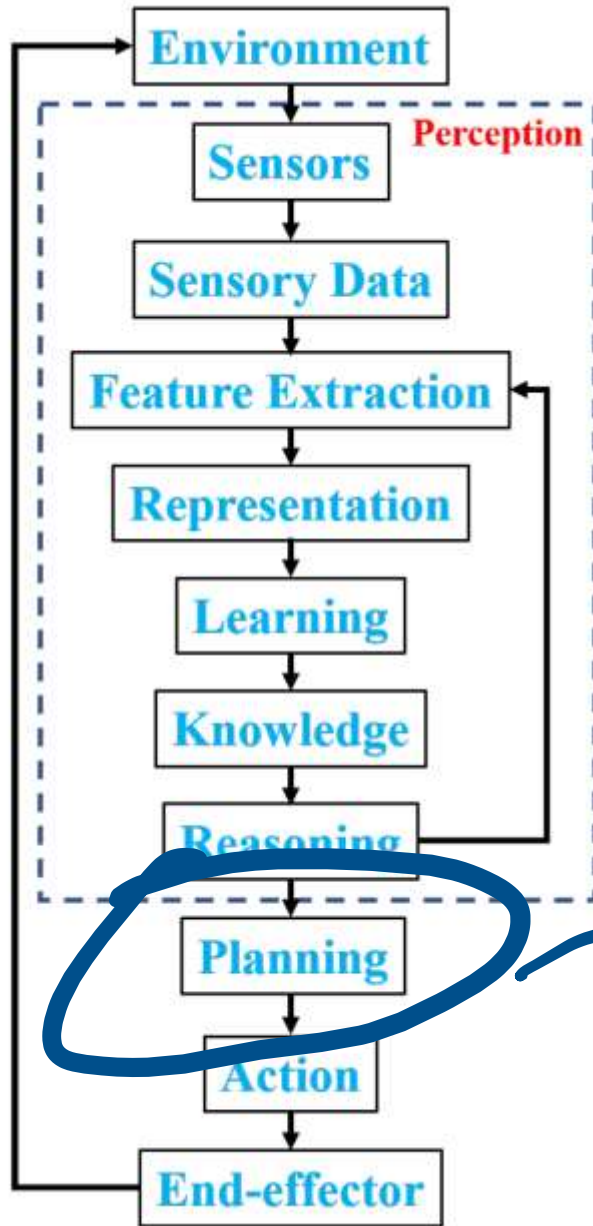


# Today's Agenda

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- Imitation learning (~20)
- Reinforcement learning (~10)



# Planning in Robotics



Robotics – Learn the **mapping** from perception to action

real-time.

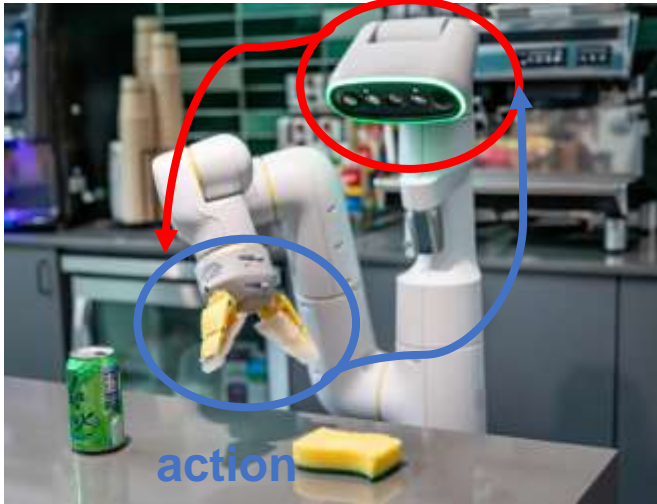
fast sampling

fast optimization

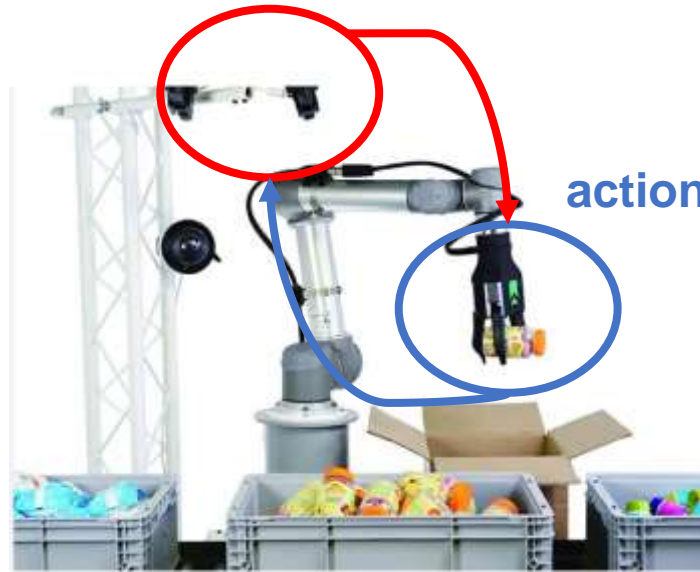


# Motion Planning in Robotics

perception



perception



action



perception

Robotics – Learn the **mapping** from perception to action

↓ not just a  $y=f(x)$



# Today's Agenda

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- **Imitation learning (~20)**
- Reinforcement learning (~10)



# Learning

## Learning Modes

**Explicit Learning:**  
Reinforcement  
Verbal instructions

**Implicit Learning:**  
Observational learning  
**Imitation learning**





# Imitation learning

Learning seems to be a negative force in evolution.  
**How can learning have evolved?**

*Learning serves as a pacemaker for evolution, when exploratory behavior leads to a breakthrough for the survival of the species, the capacity for that kind of exploratory behavior and the imitation of this act is favored by natural selection.*



# Imitation learning

## Imitation Capabilities in Animals

Which species may exhibit imitation is still a main area of discussion and debate

One differentiate “true” imitation from copying (flocking, schooling, following), stimulus enhancement, contagion or emulation

Biological  
Inspiration





# Imitation learning

$$\vec{x} = \vec{x}'$$

Same Object, same target location

$$\vec{d} = \vec{d}'$$

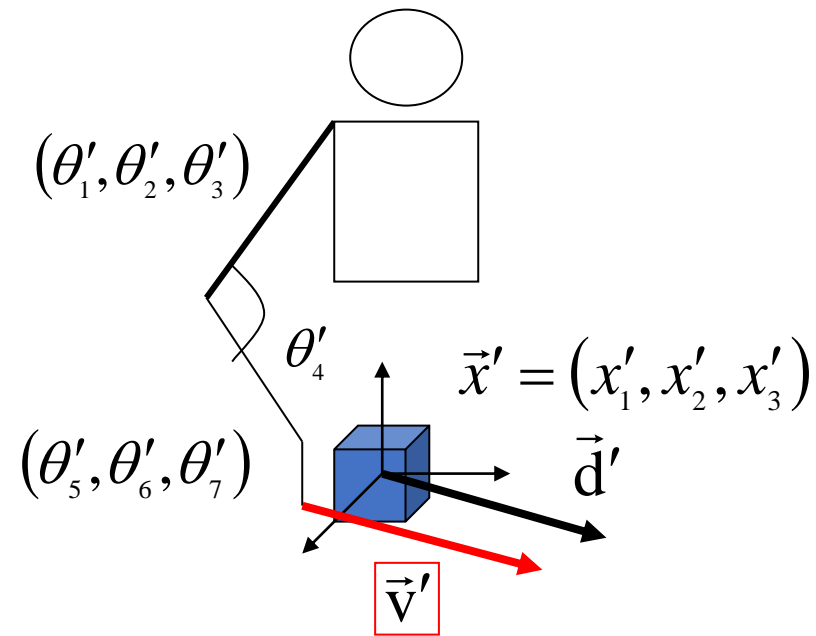
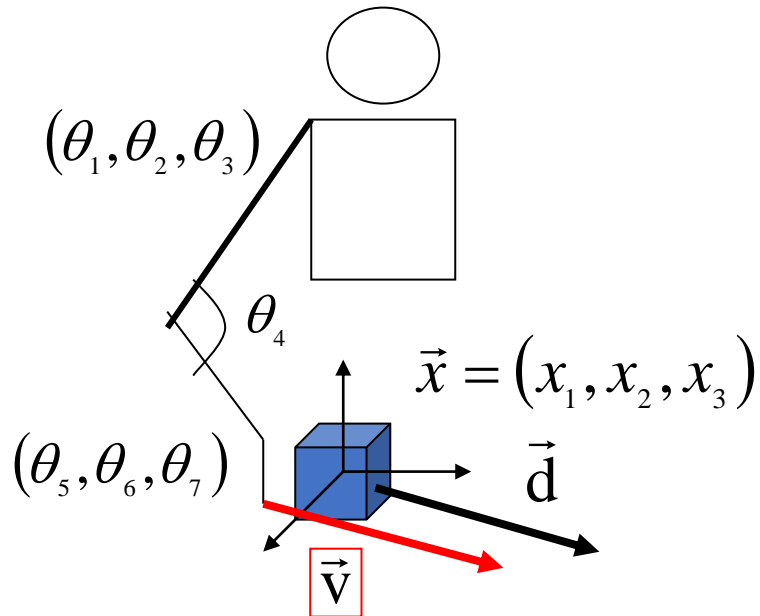
Same direction of motion

$$\vec{v} = \vec{v}'$$

Same speed, same force

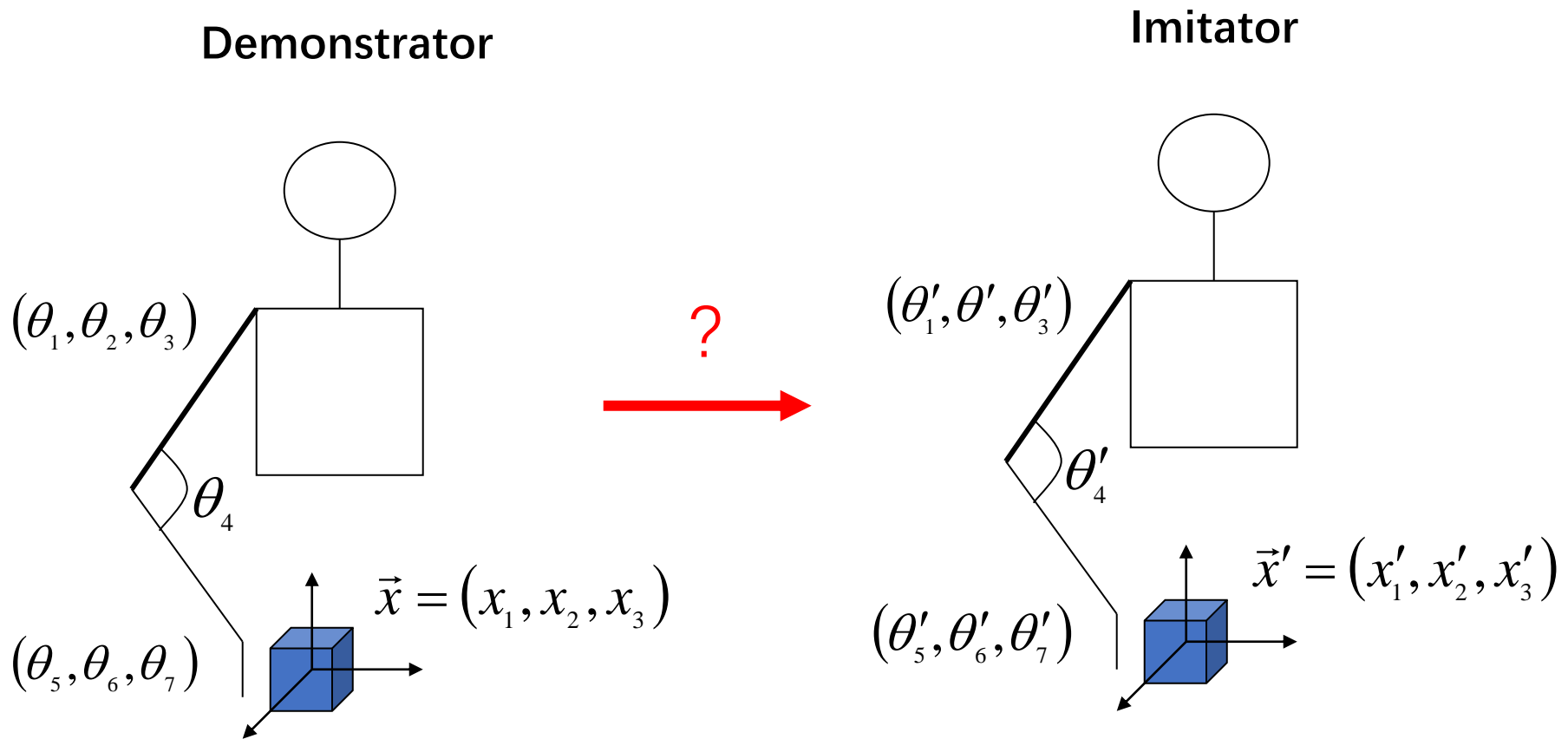
$$\vec{\theta} = \vec{\theta}'$$

Same posture





# Imitation learning

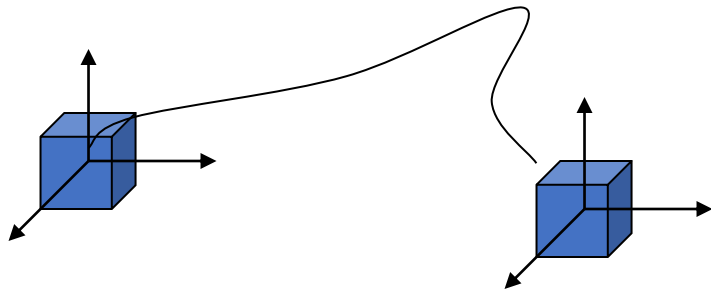


The Transfer problem



# Imitation learning

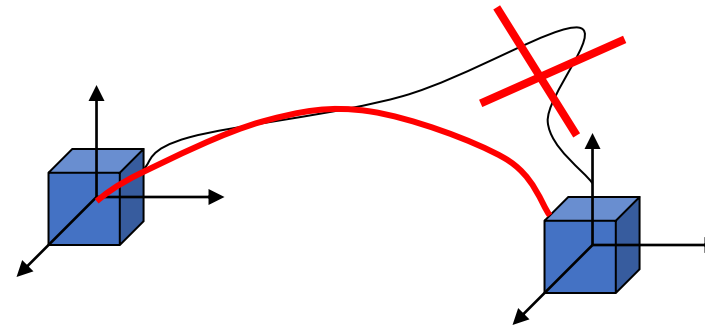
Demonstration



?



Imitation



**No solutions** (smaller range of motion)

→ Find the closest solution according to a metric

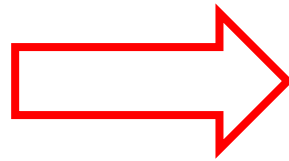
**How to Imitate?**

**The correspondence problem**



# Imitation learning

*Learning What to imitate*



Imitation learning – Programming by Demonstration:

- A way **to speed up learning**, to reduce the search space
- A way **to share** with robots the same **vocabulary of motor skills**



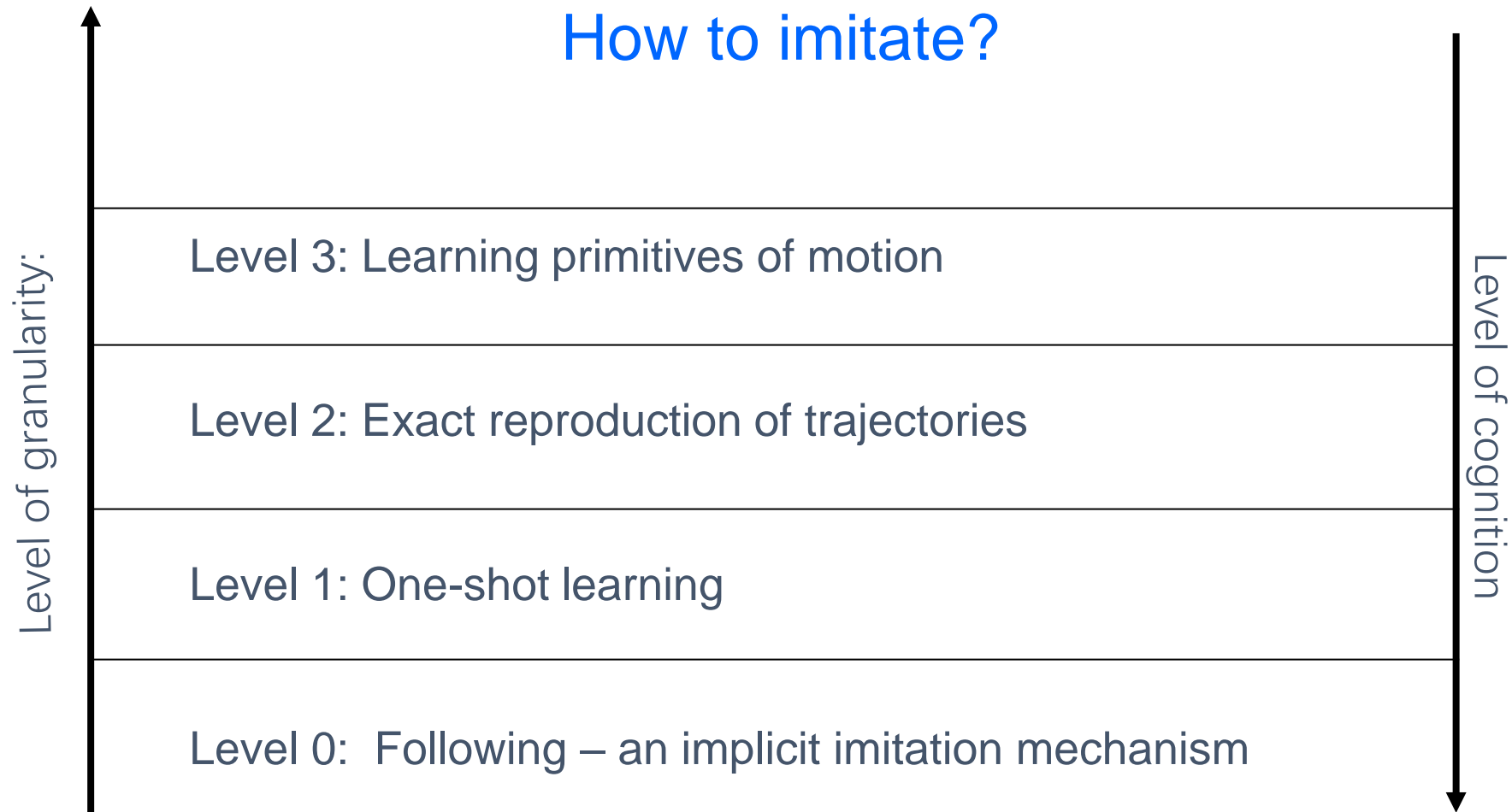
# Imitation learning

Imitation Learning in Robots  
Prof. Aude Billard, [lisa.epfl.ch](http://lisa.epfl.ch)

Granularity

Cognition

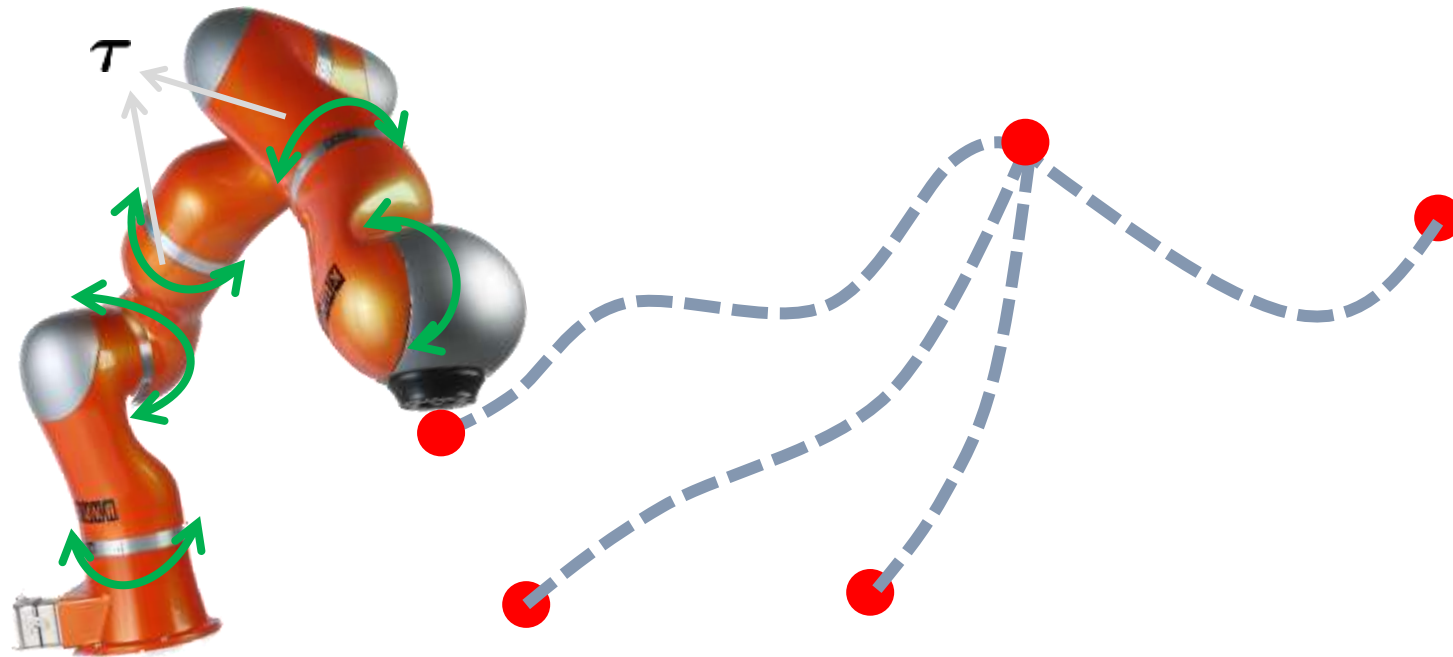
How to imitate?





# Imitation learning

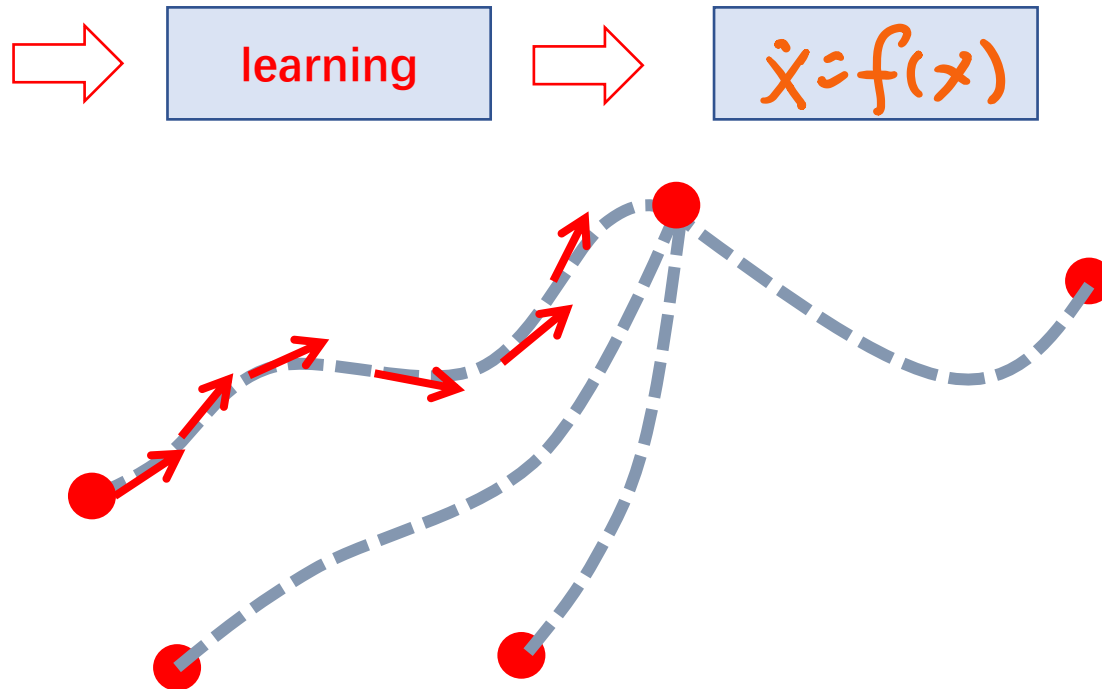
$$\mathbf{M}_h(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}}_r + \mathbf{C}_h(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \mathbf{g}_h(\boldsymbol{\theta}) = \boldsymbol{\tau}$$







# Imitation learning





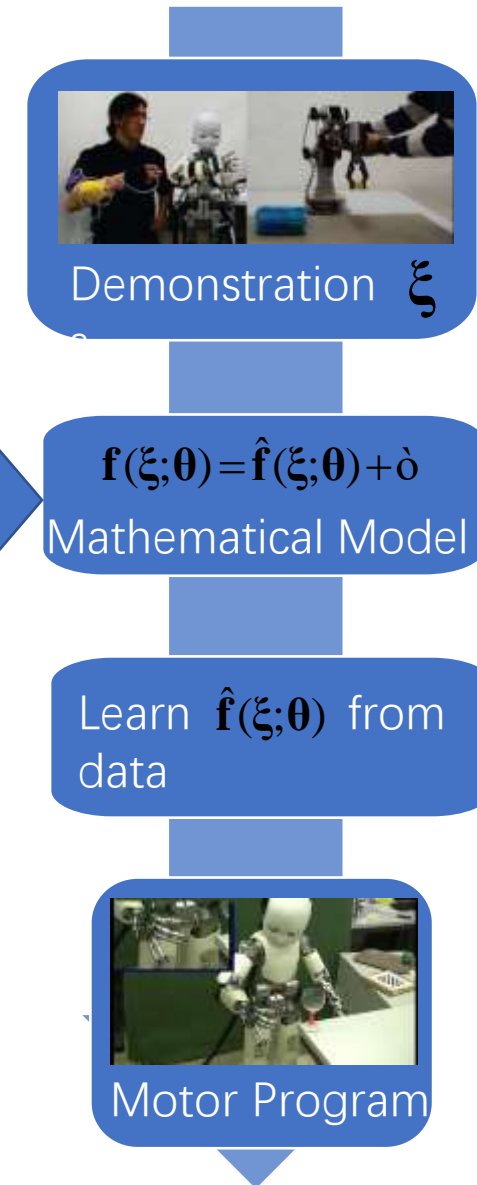
# Imitation learning

## *Programming by Demonstration (Imitation Learning)*

- ▶ A task is characterized by an underlying deterministic relationship between the **relevant** variables

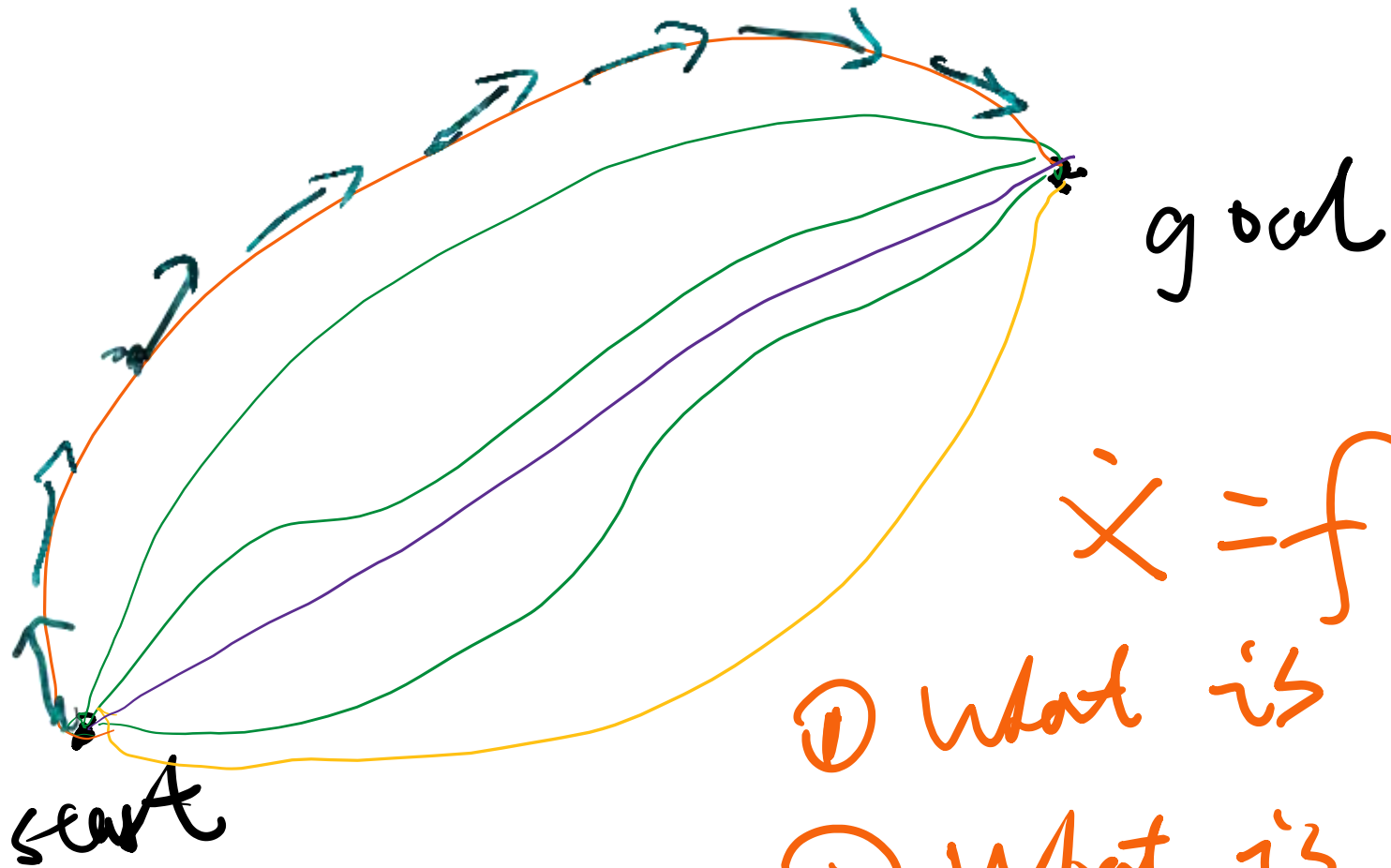


- ▶ Demonstration: reproducing the underlying relationships corrupted by **white** noise.





# Imitation learning



$$\dot{x} = f(x)$$

- ① what is  $x$ ?
- ② what is  $f$ ?

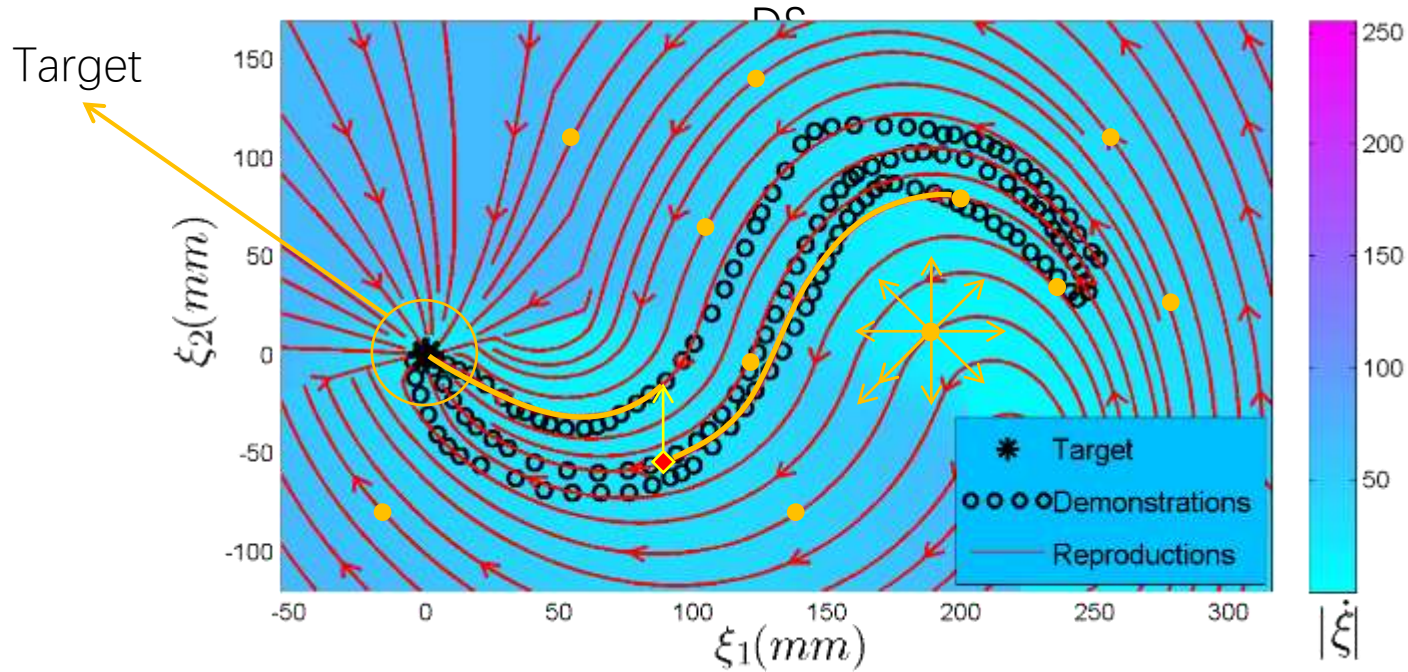
Question: collision?



# Imitation learning

$$\dot{\xi} = f(\xi)$$

Streamlines of a *globally asymptotically stable*



Given: Some demonstrations of a point-to-point motion.

Learned: Globally asymptotically stable map from states to velocities stable at the sole target.



# Imitation learning

exp design:

hardware  
sensor  
protocol  
intention  
interface

data collection:

joint angles  
pos / ori  
force  
tactile-  
vision  
⋮

Learning Alg

GMM  
GP  
SVM  
⋮  
Deep learning  
LLM  
RT-2.



# Imitation learning

exp design.

data collection:

Learning Alg

hardware

joint angles

GMM

**More details will be introduced in imitation learning course!**

Sens

protocol

intention

interface

tactile-  
vision

⋮

M

⋮

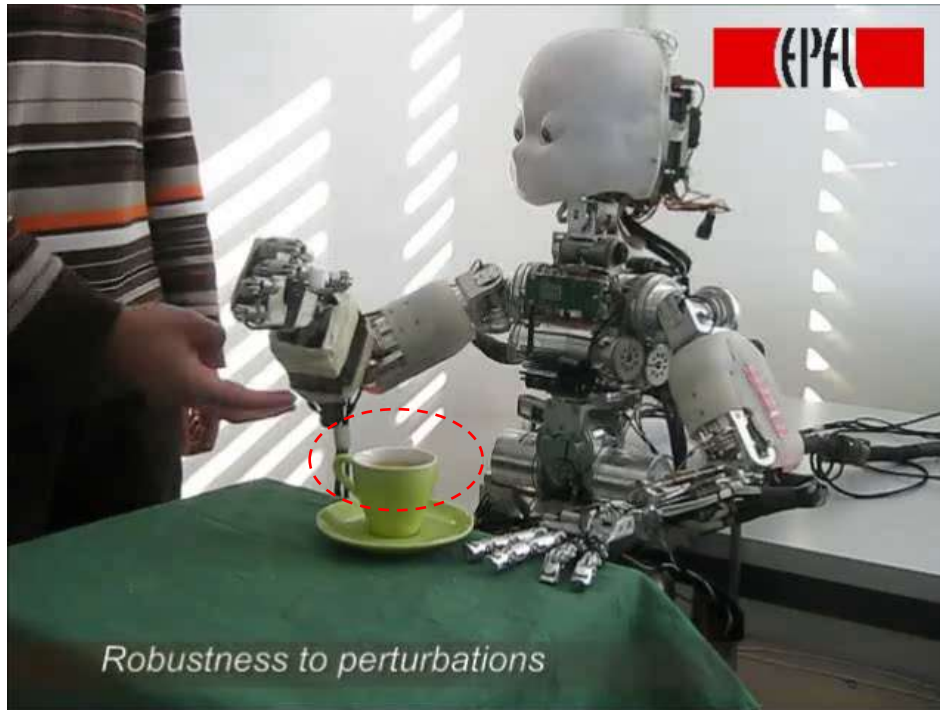
Deep learning

LLM

RT-2.



# Imitation learning





# Imitation learning

## Google Deep Learning for Grasping

Learning Hand-Eye Coordination for Robotic Grasping with Deep Learning  
and Large-Scale Data Collection

Sergey Levine  
Peter Pastor  
Alex Krizhevsky

SLEVINE@GOOGLE.COM  
PETERPASTOR@GOOGLE.COM  
AKKRIZHEVSKY@GOOGLE.COM

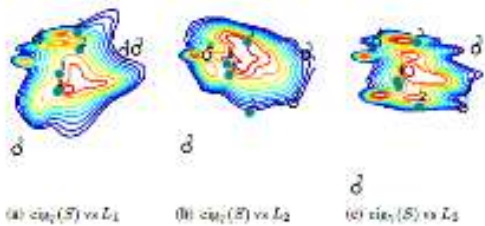
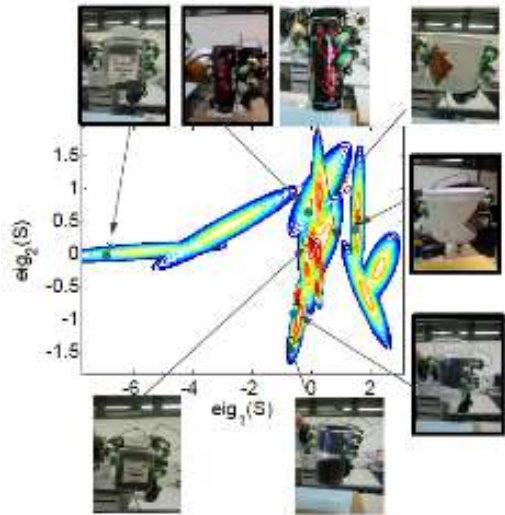




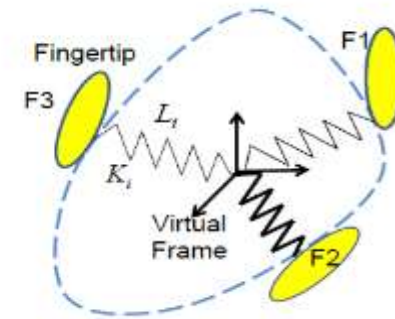


# Imitation learning

Stability = sensory information  
+ motor action



## Object-level Impedance Controller



Learning of Grasp Adaptation through  
Experience and Tactile Sensing

Miao Li, Yasemin Bekiroglu,  
Danica Kragic and Aude Billard

IROS 2014



# Imitation learning

Student project. intro  
1. ultrasound Robot  
2. CraspAda.



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- **Imitation learning (~20)**
- **Reinforcement learning (~10)**



# Goal for this course

- **Design: soft hand design x1**
- **Perception: vision, point cloud, tactile, force/torque x1**
- **Planning: sampling-based, optimization-based, learning-based x3**
- **Control: feedback, multi-modal x2**
- **Learning: imitation learning, RL x2**
- **Simulation tool (pybullet, matlab, OpenRAVE, Issac Nvidia, Gazebo)**
- **How to get a robot moving!**