



Robotics

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Fall 2023, Wuhan University

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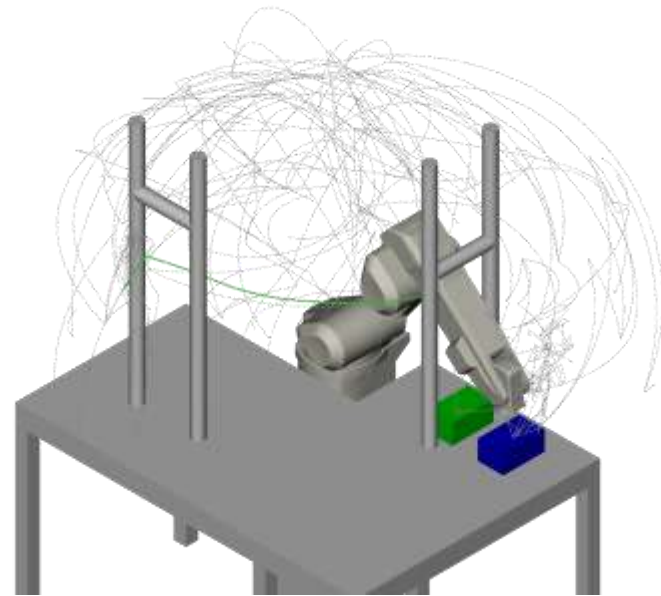
Goal for this course

- Design: soft hand design **x1**
- Perception: vision, point cloud, tactile, force/torque **x1**
- Planning: sampling-based, optimization-based, learning-based **x3**
- Control: feedback, multi-modal **x2**
- Learning: imitation learning, RL **x2**
- Simulation tool (pybullet, matlab, OpenRAVE, Issac Nvidia, Gazebo)
- **How to get a robot moving!**



Today's Agenda

- **What is planning? (~10)**
- **Motion planning in robotic application (~10)**
 - Self-driving, drone, robot arm, humanoids, medical robots, soft robots ...
- **Formulation of robot motion planning**
- **Planning as searching (~25)**
- **Planning as sampling (~25)**
 - PRM, RRT, RRT*

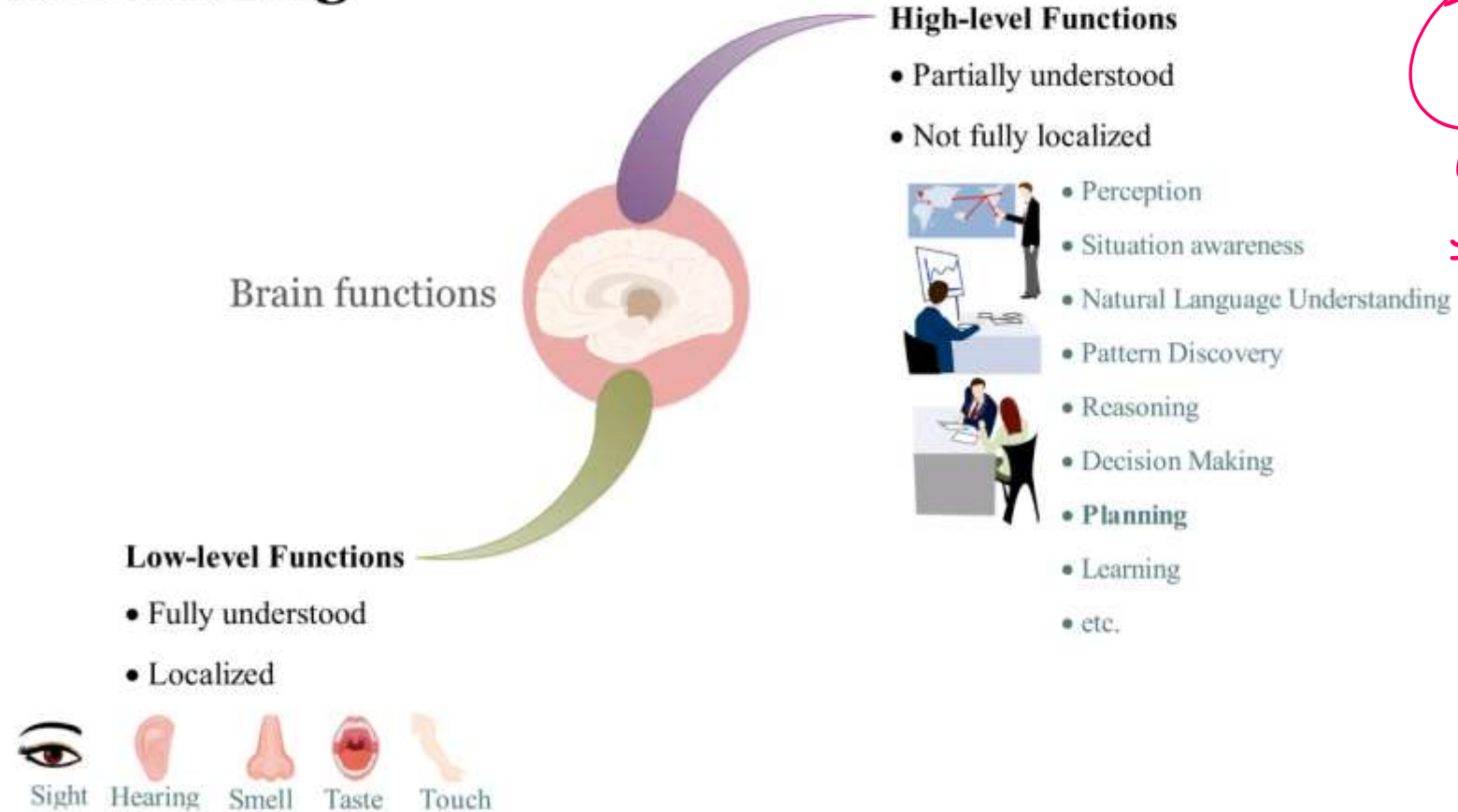




What is planning?

Q: How can I get there from here?

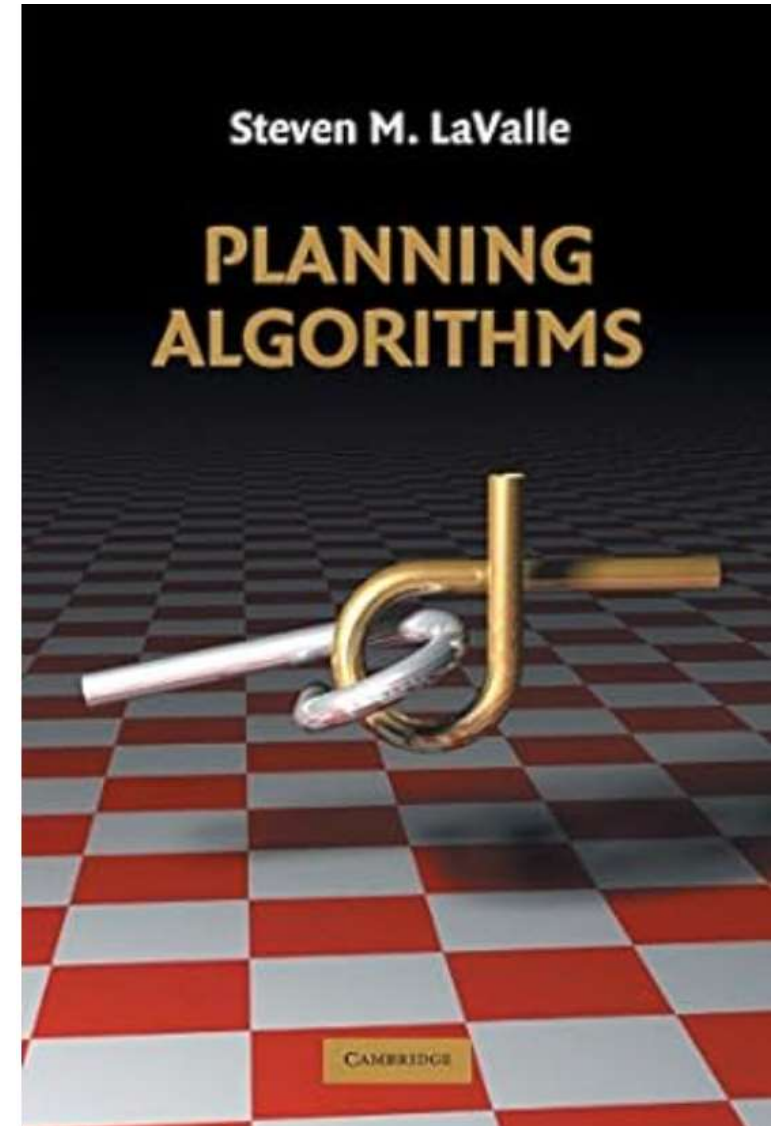
A: Planning



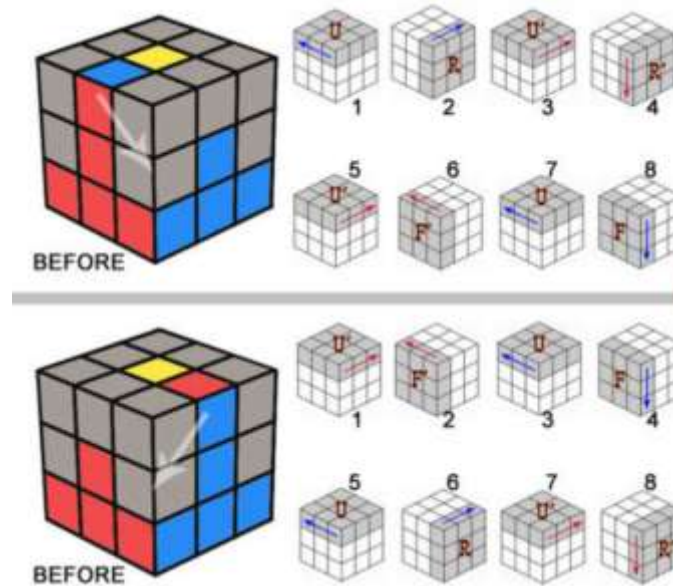
initial state
↓
? goal state



What is planning?



| | | | |
|----|----|---|----|
| 張飛 | 曹操 | | 趙雲 |
| 馬超 | 關羽 | | 黃忠 |
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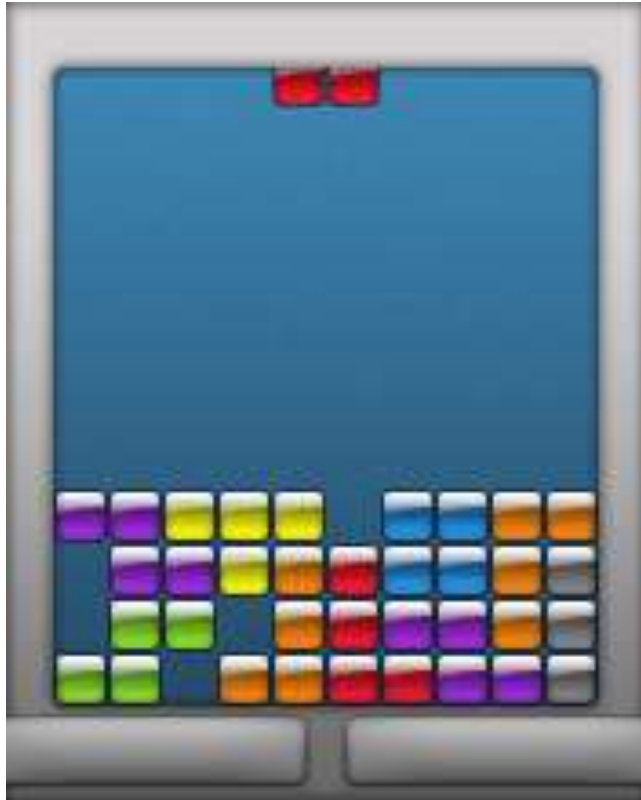


What is planning?



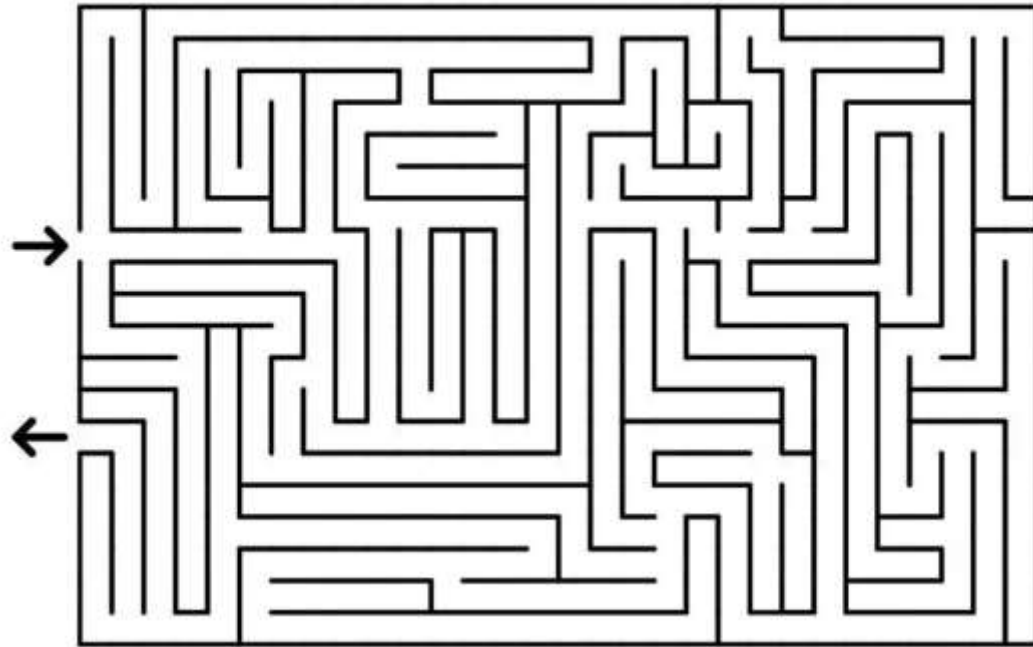
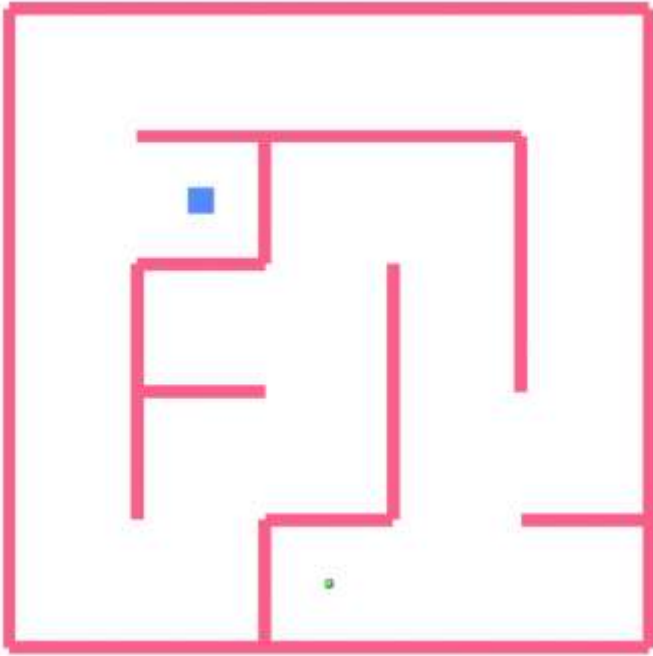


What is planning?



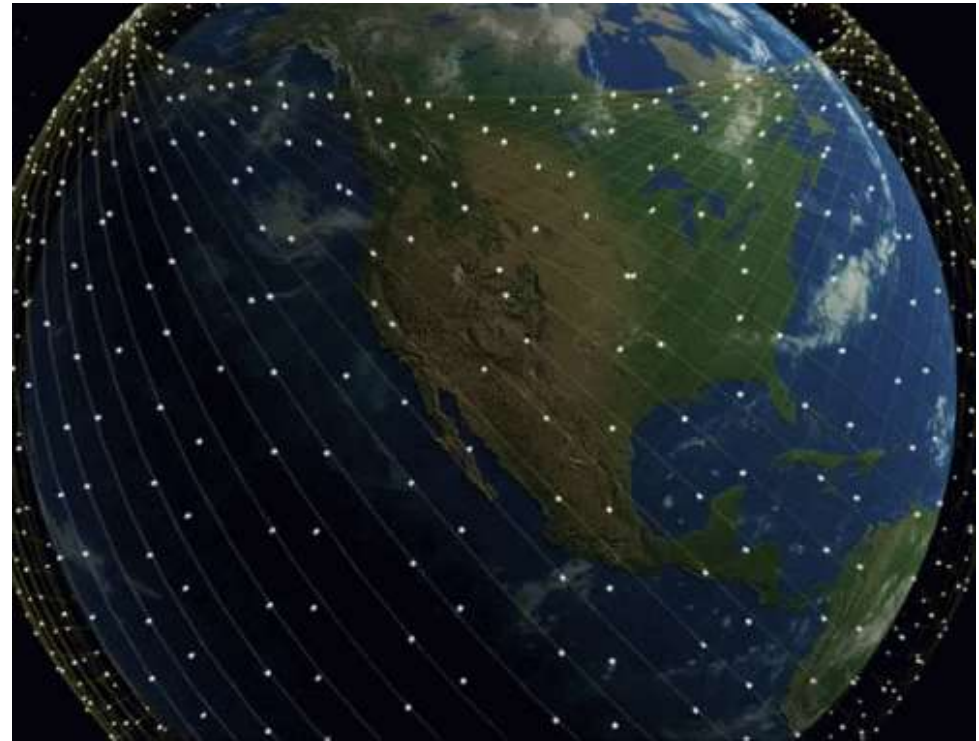
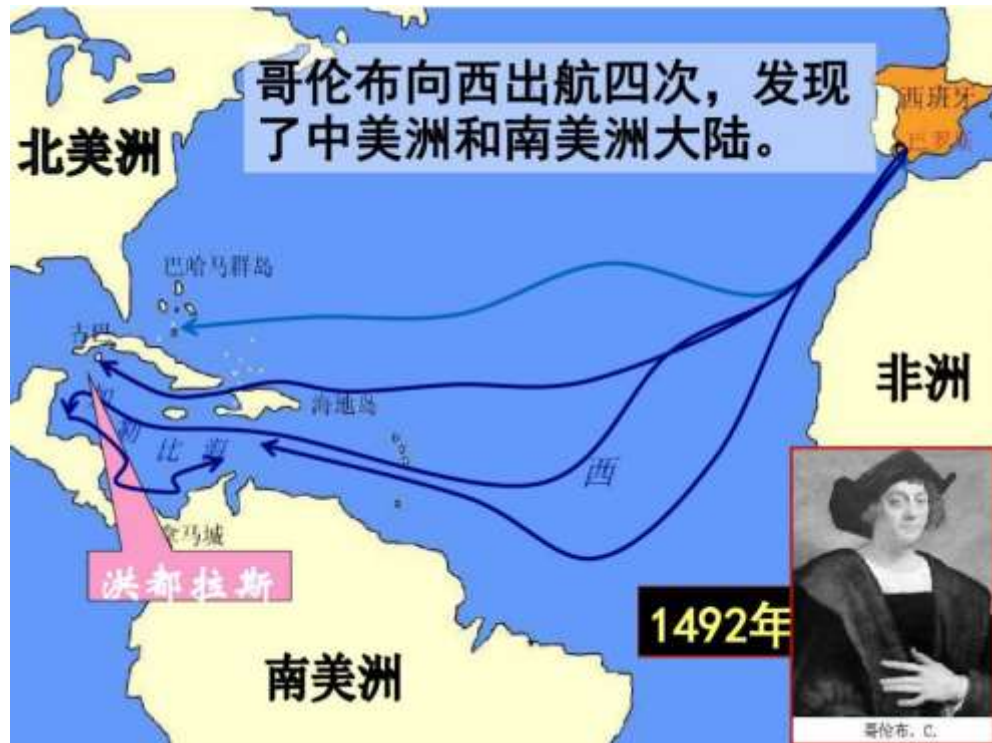


What is planning?





What is planning?





What is planning?





What is planning?





What is planning?





What is planning?





What is planning?

A **plan** is typically any diagram or list of steps with details of timing and resources, used to **achieve an objective to do something**. It is commonly understood as a temporal set of intended actions through which one expects to achieve a goal.

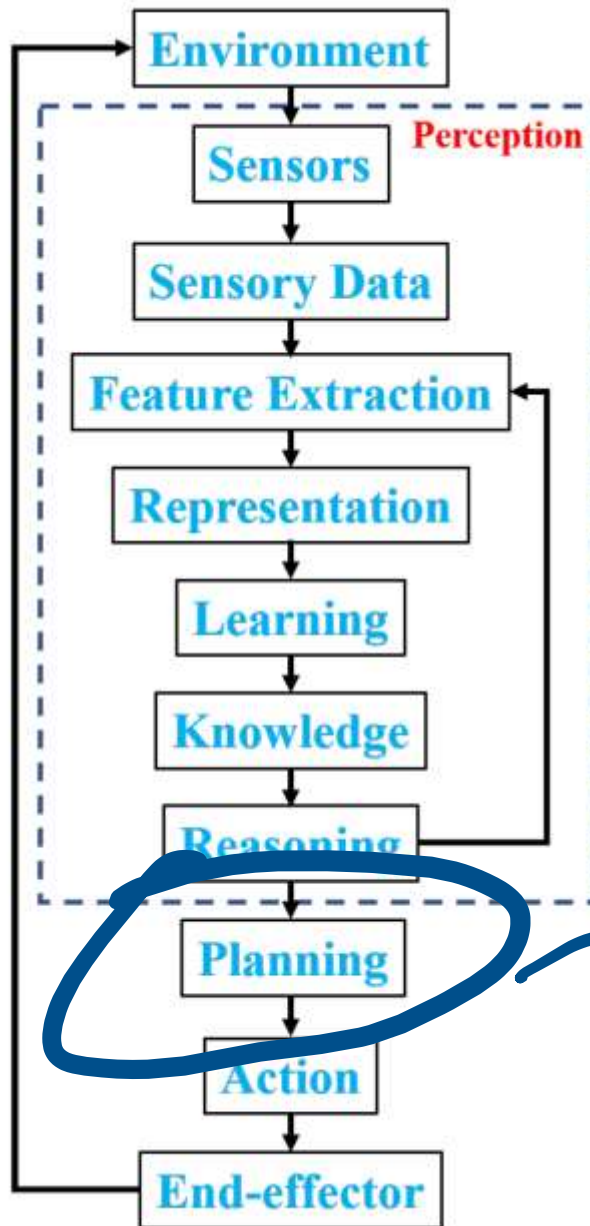


What is planning?

Any other interesting
examples?



Planning in Robotics



Robotics – Learn the **mapping** from perception to action

real-time.

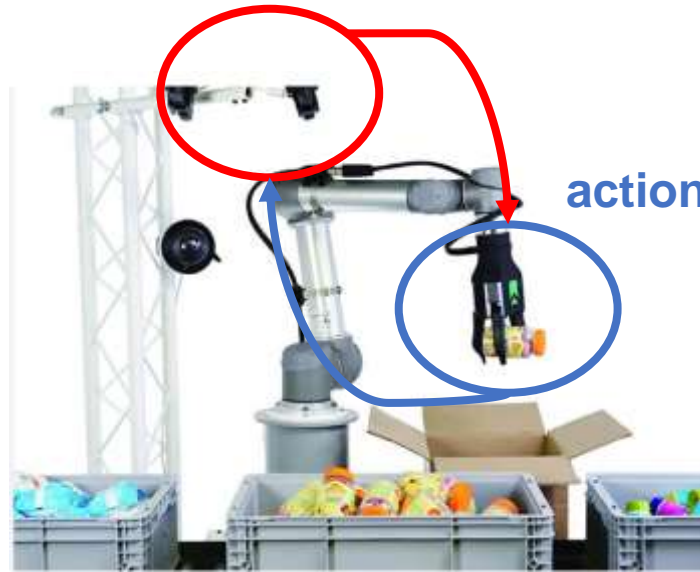


Motion Planning in Robotics

perception



perception



action



perception

Robotics – Learn the **mapping** from perception to action



Motion Planning in Robotics



Self-driving



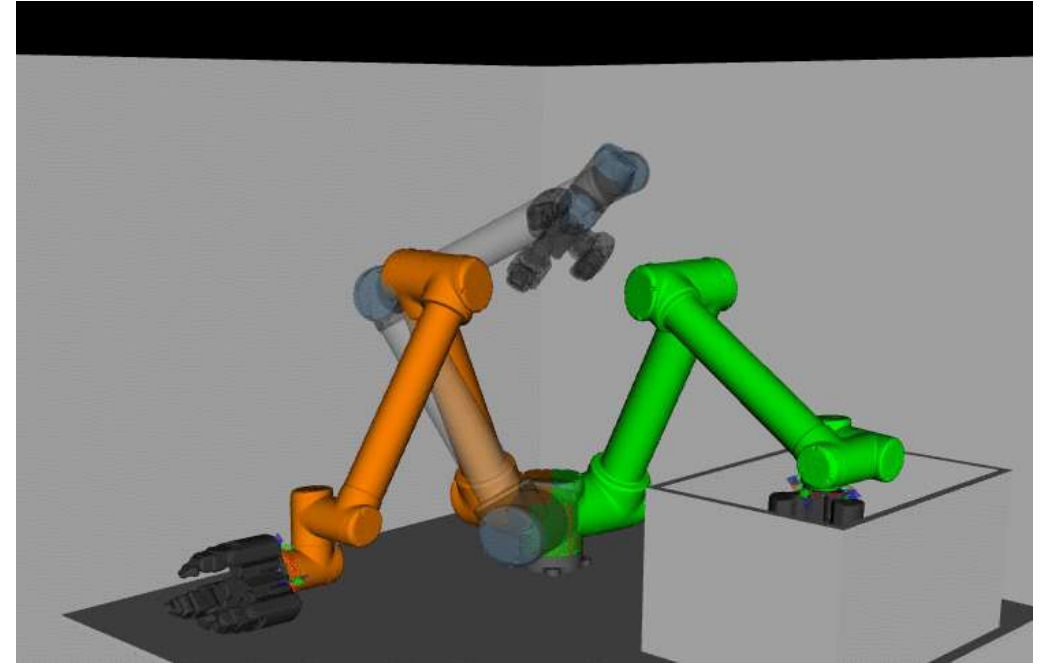
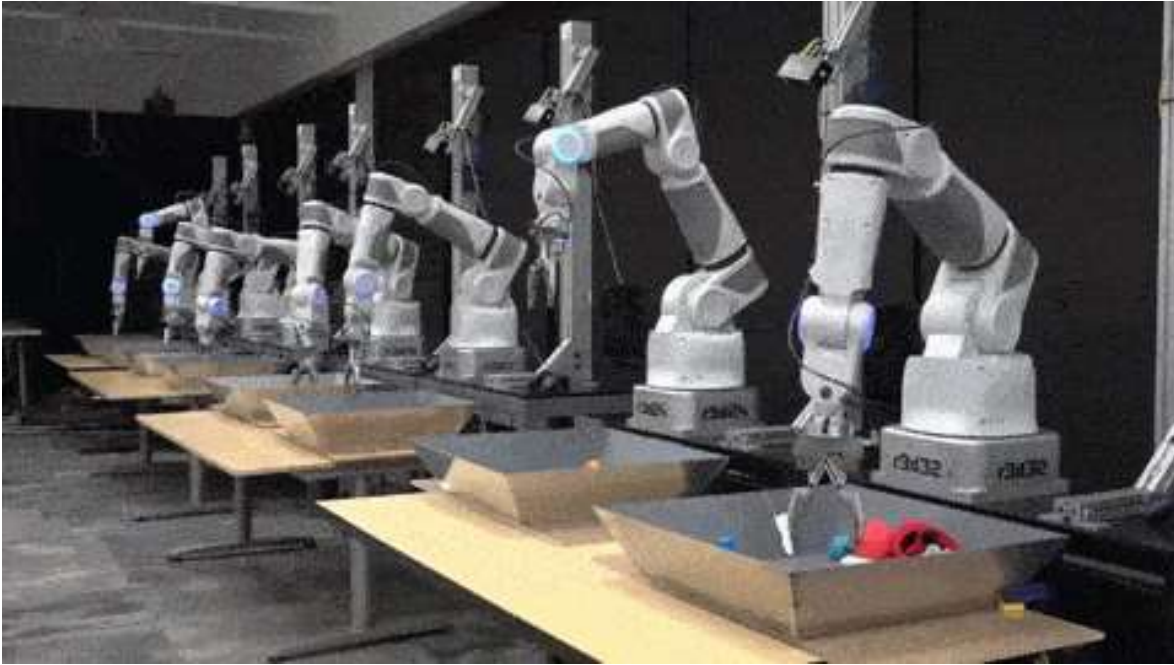
Motion Planning in Robotics



Drones



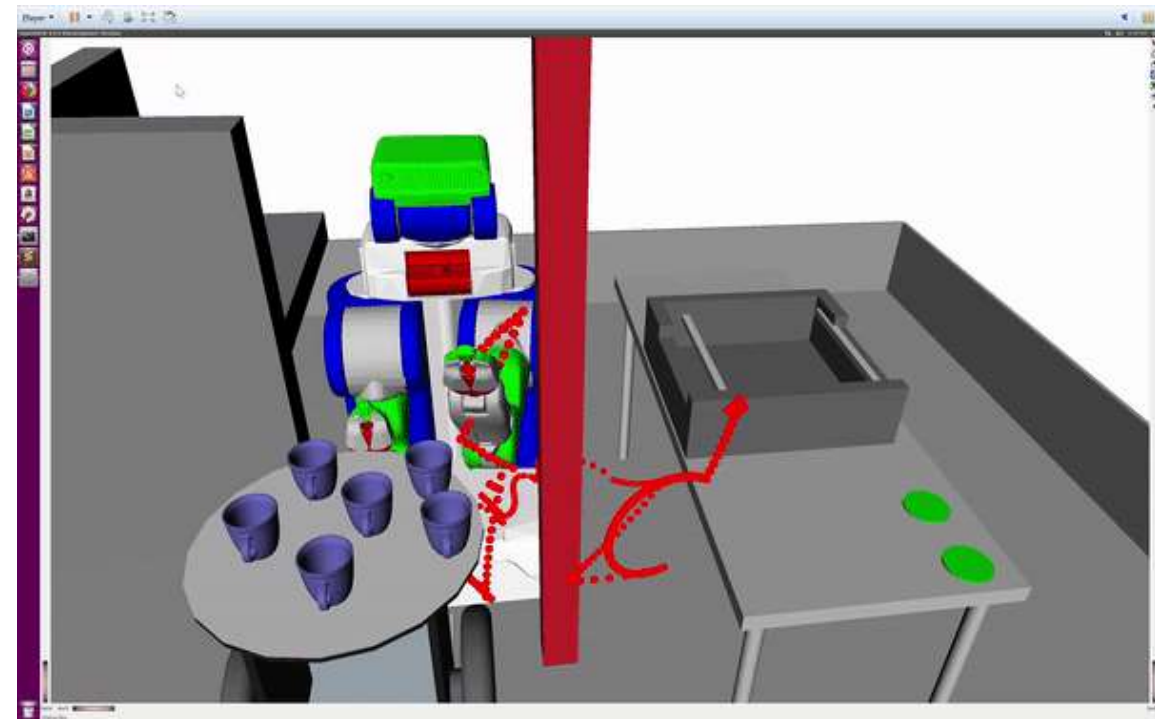
Motion Planning in Robotics



Robot arms



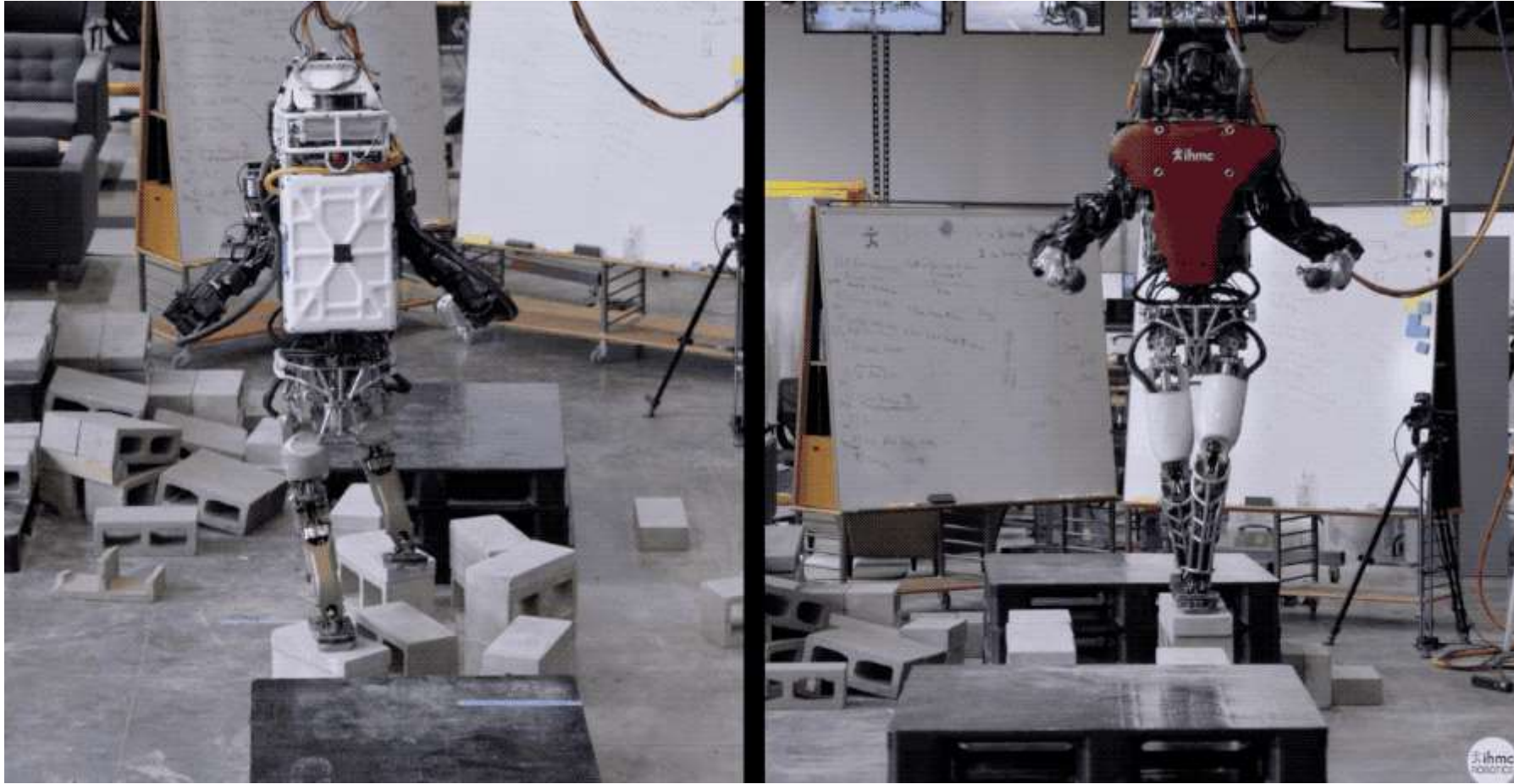
Motion Planning in Robotics



Bimanual manipulation



Motion Planning in Robotics



Humanoids



Motion Planning in Robotics



Humanoids



Motion Planning in Robotics



Tesla's Optimus Robot Sort Objects Autonomously

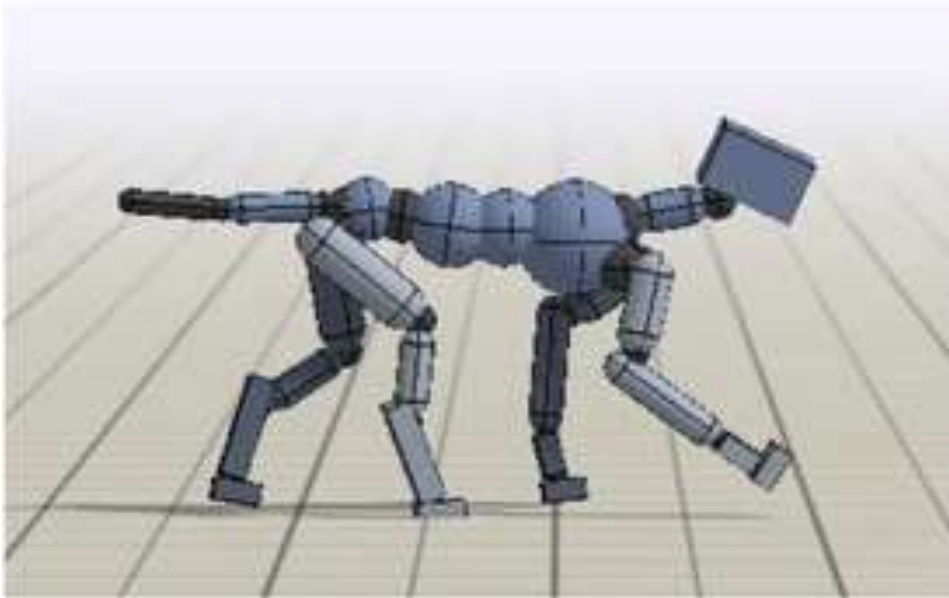
https://www.youtube.com/watch?v=oL5YNtDUQXU&ab_channel=CNETHighlights

Humanoids

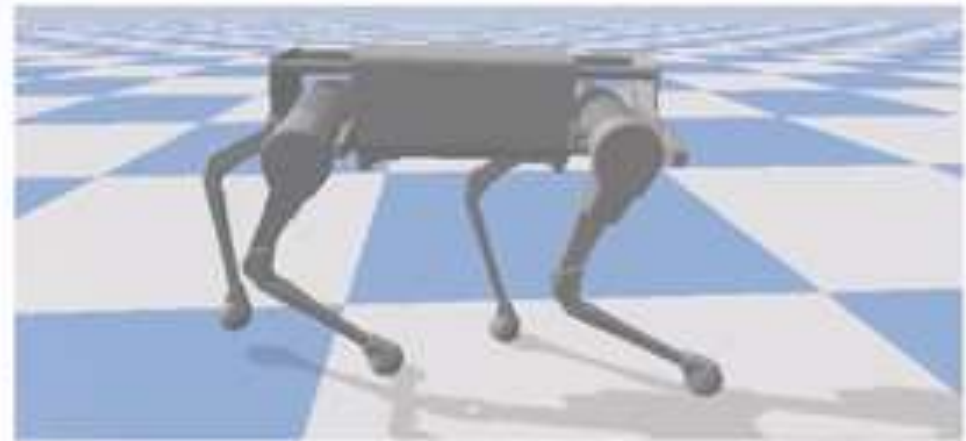


Motion Planning in Robotics

Dog Pace



Mocap Data



Reference Motion

Quadruped robot



Motion Planning in Robotics



Quadruped robot



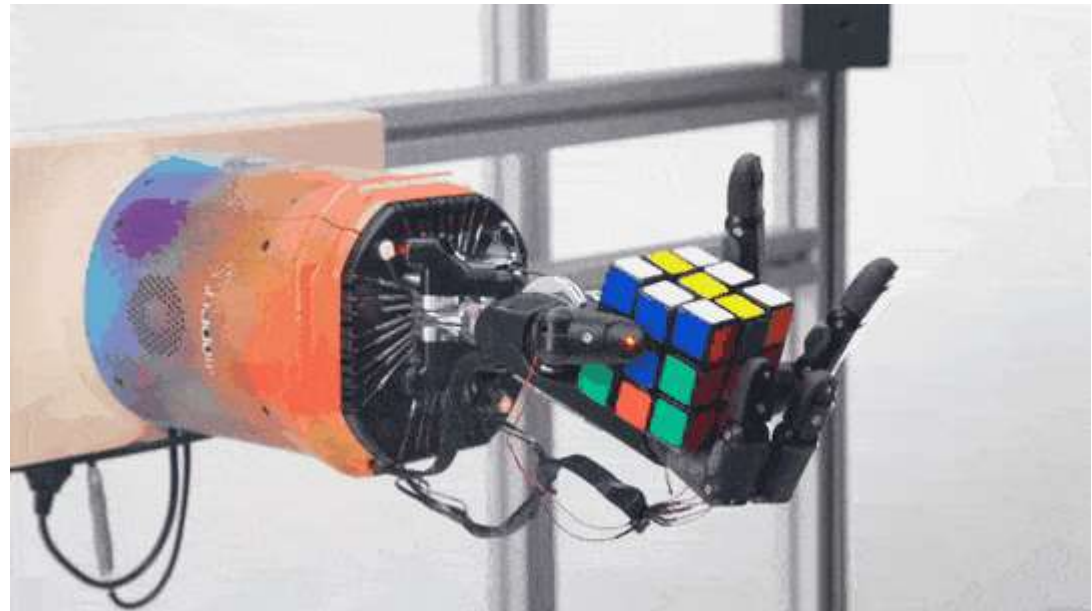
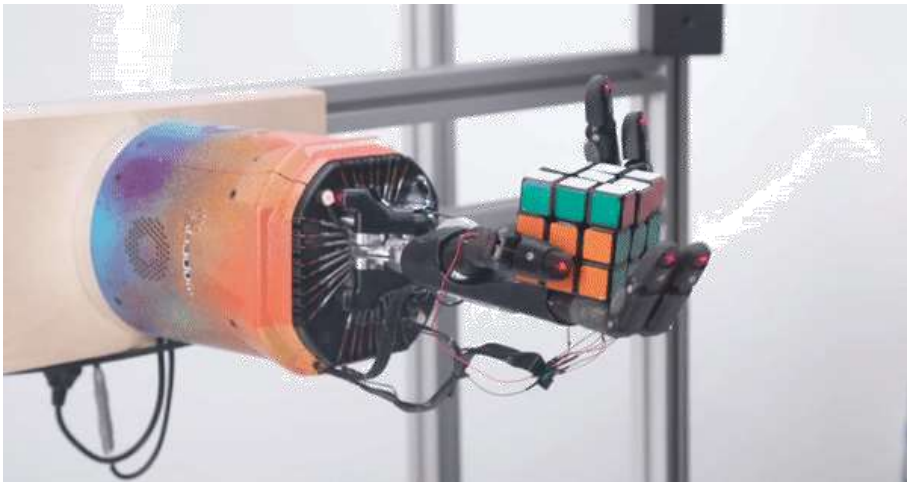
Motion Planning in Robotics



Medical robot



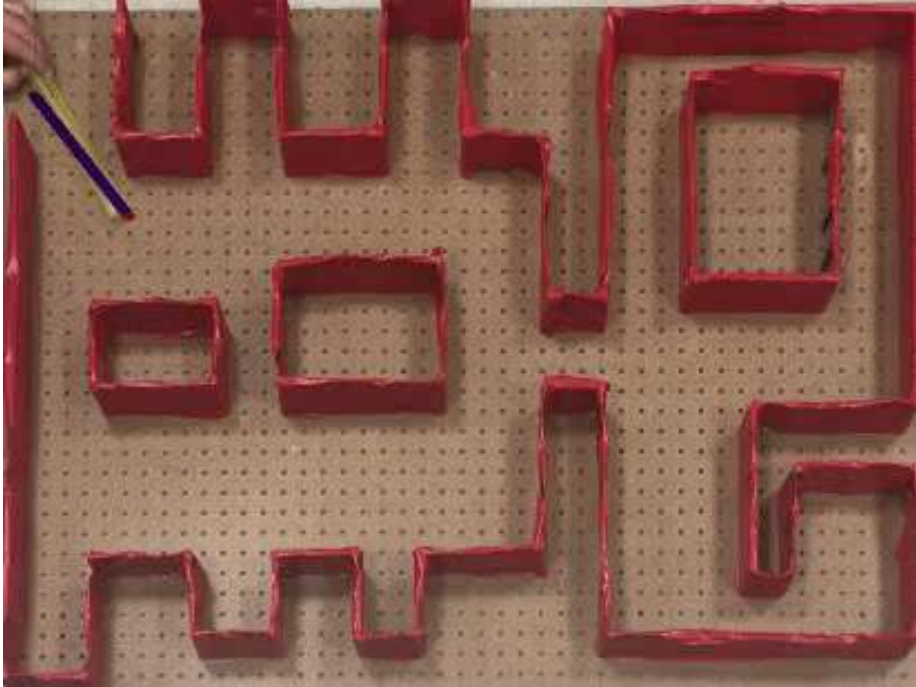
Motion Planning in Robotics



Dexterous manipulation



Motion Planning in Robotics

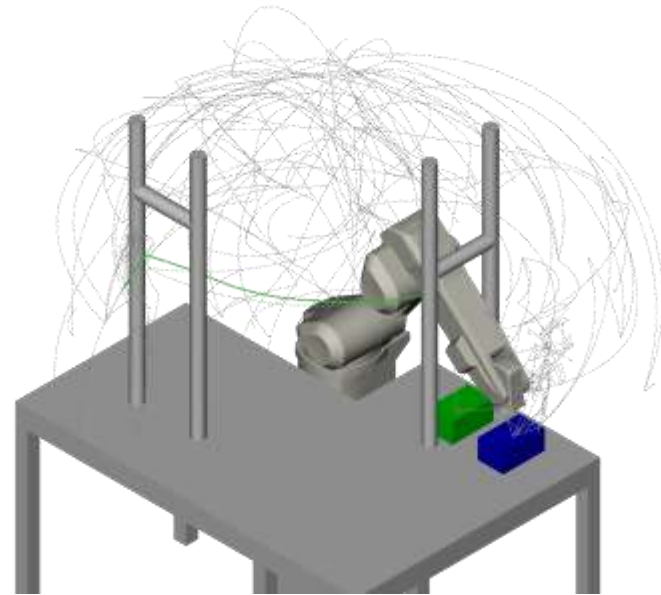


Soft robots



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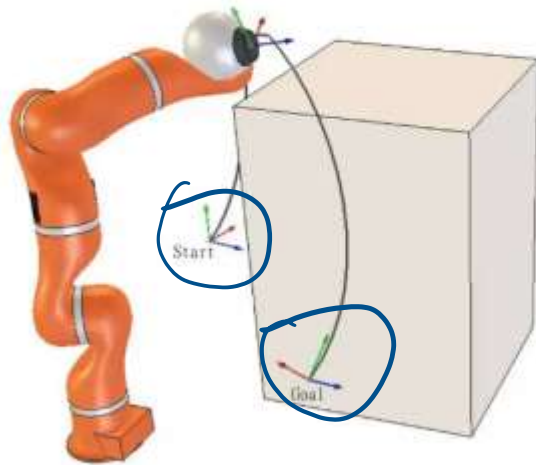




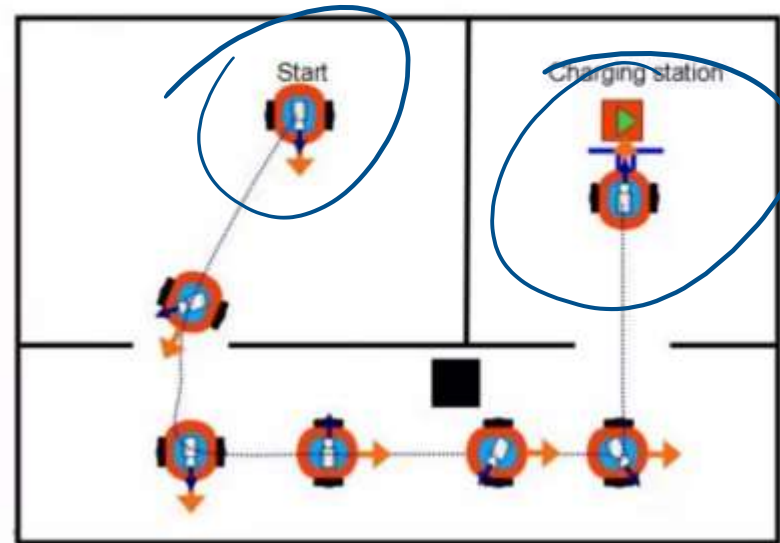
Robot Motion Planning

- Motion planning is a term used in robotics for the process of breaking down a desired **movement task** into **discrete motions** that satisfy movement **constraints** and possibly **optimize** some aspect of the movement.

A robot has to compute a **collision-free path** from a start position (s) to a given goal position (G), amidst a collection of obstacles.



Articulated Robot

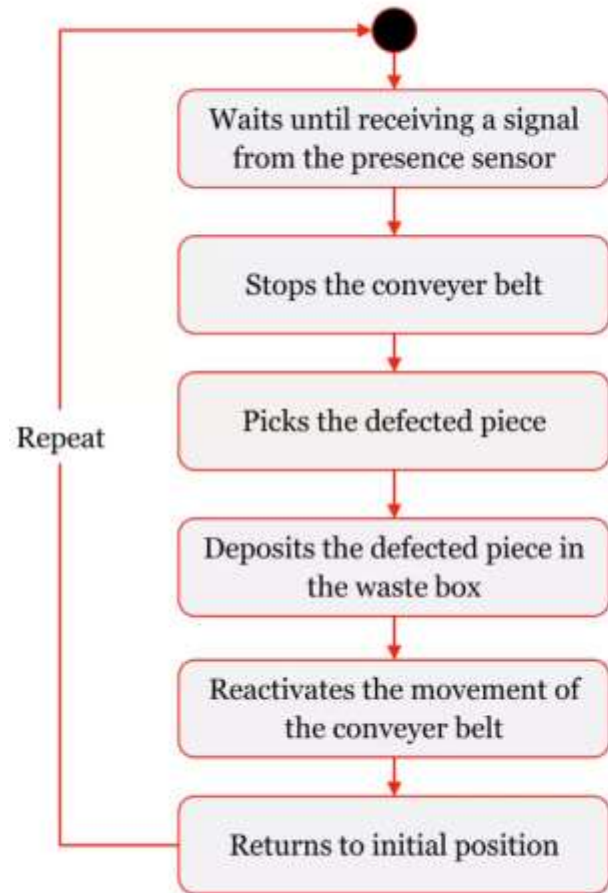


Rigid Robot

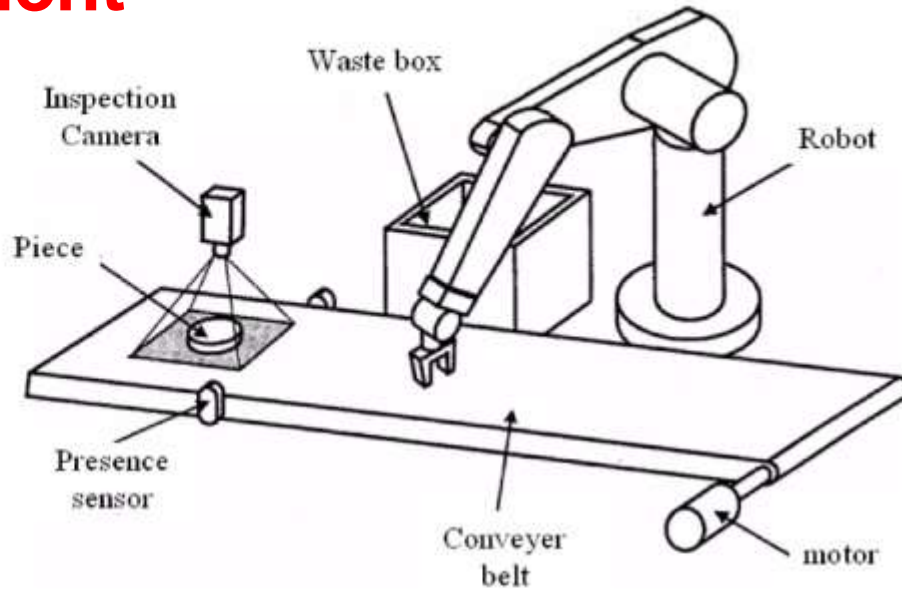


Robot Motion Planning

Structured environment



Plan: Activity Diagram

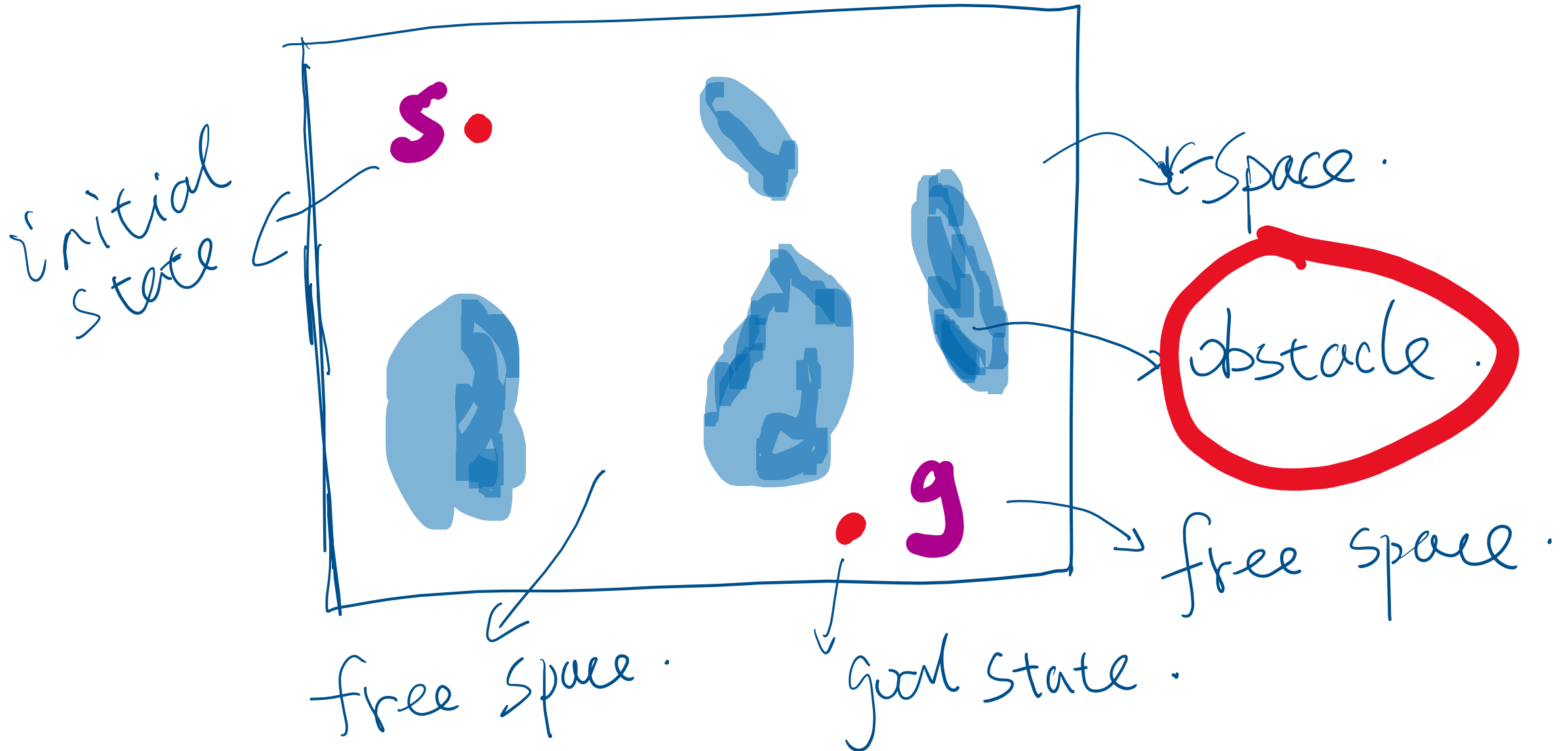


```
PROC main()
  go_wait_position;           !Move to initial position
  WHILE Dinput(finish)=0 Do  !wait end program signal
    IF Dinput(defected_piece)=1 THEN !Wait defected piece signal
      SetDO activate_belt, 0;      !Stop belt
      pick_piece                   !Pick the defected piece
      SetDO activate_belt, 1;      !Activate the belt
      place_piece                   !Place the defected piece
      go_wait_position;           !Move to the initial position
    ENDIF
  ENDWHILE
ENDPROC
```

ABB RAPID Program

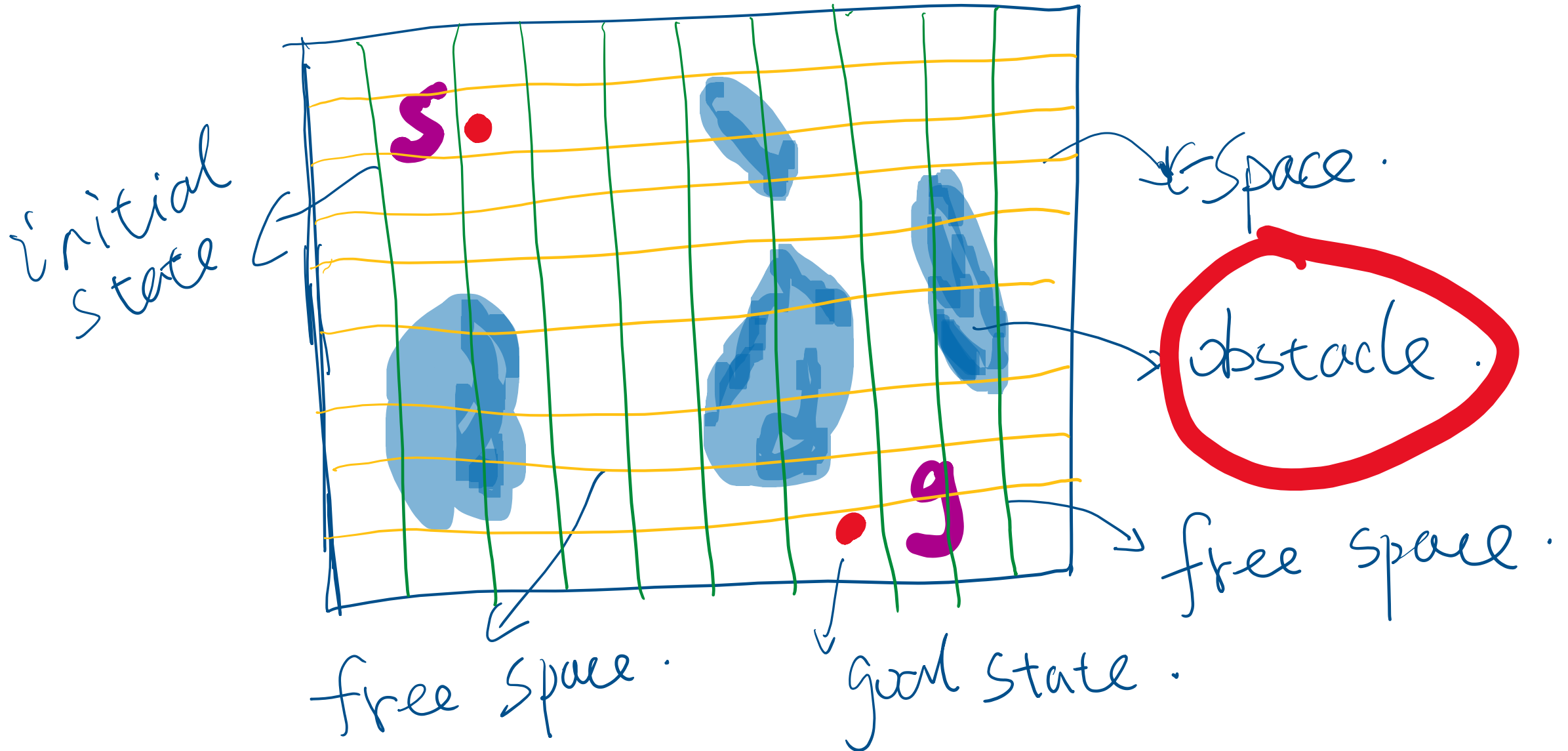


Motion Planning in 2D



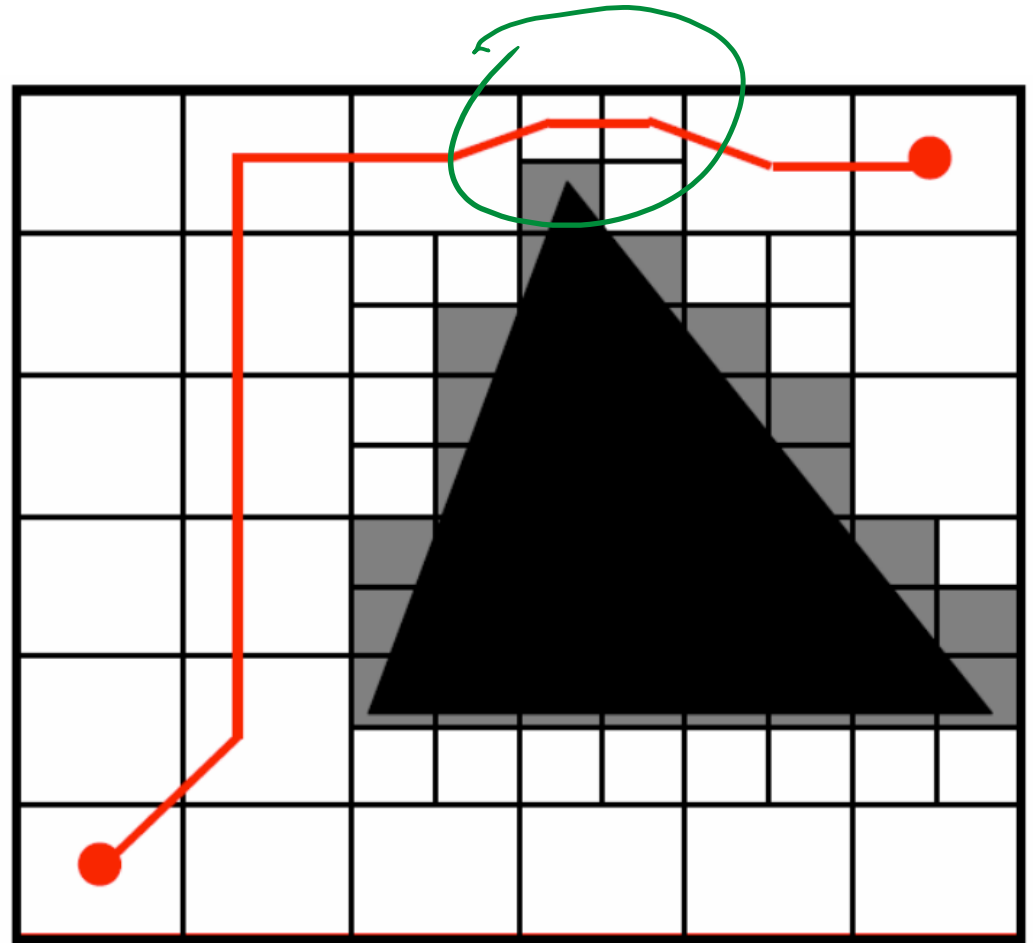
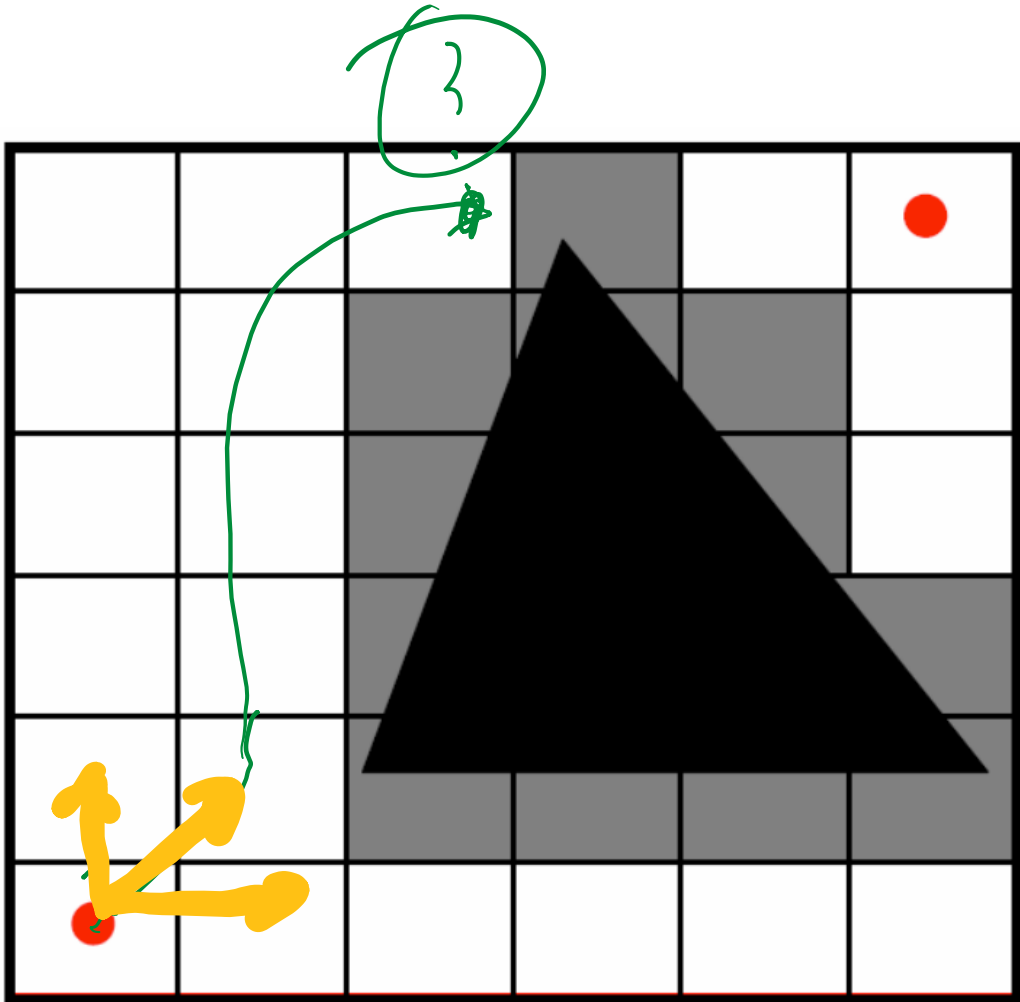


Motion Planning in Grid World





Motion Planning in Grid World



Try to find a path using the current set of cells

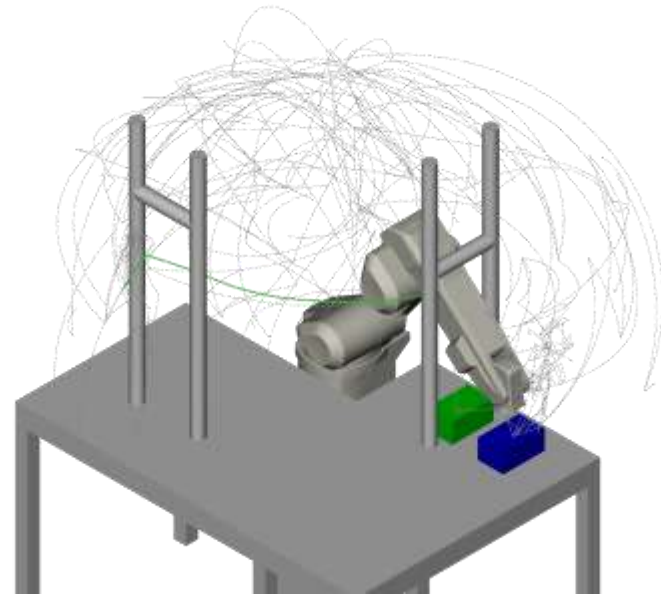
If no path found:

- Subdivide the *MIXED* cells and try again with the new set of cells



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- **Planning as sampling (~25)**
 - PRM, RRT, RRT*





Discrete Planning

Blind.

- **Breadth-First Search**
- **Depth-First Search**
- **Brute-Force Search**

Informed

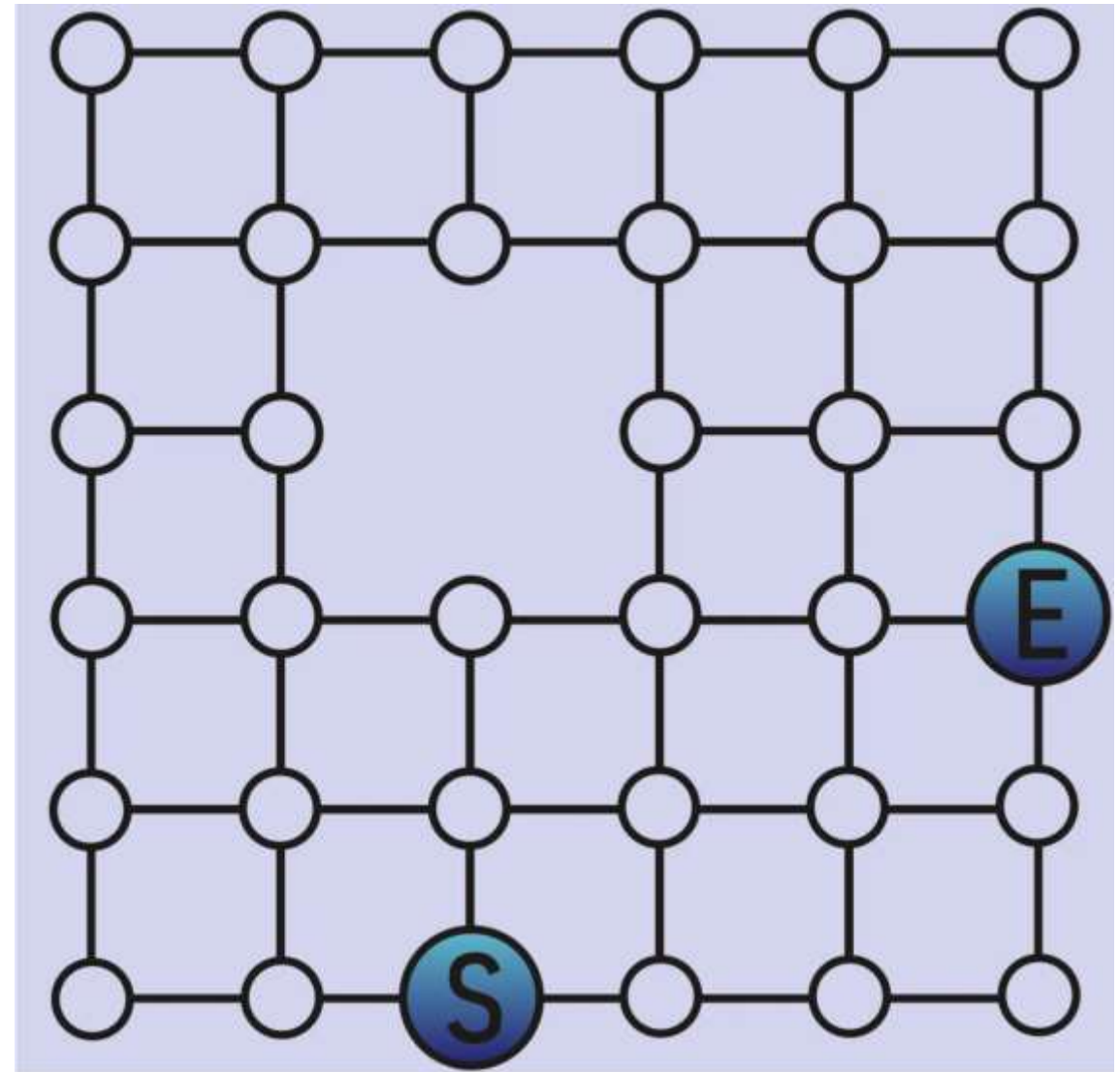
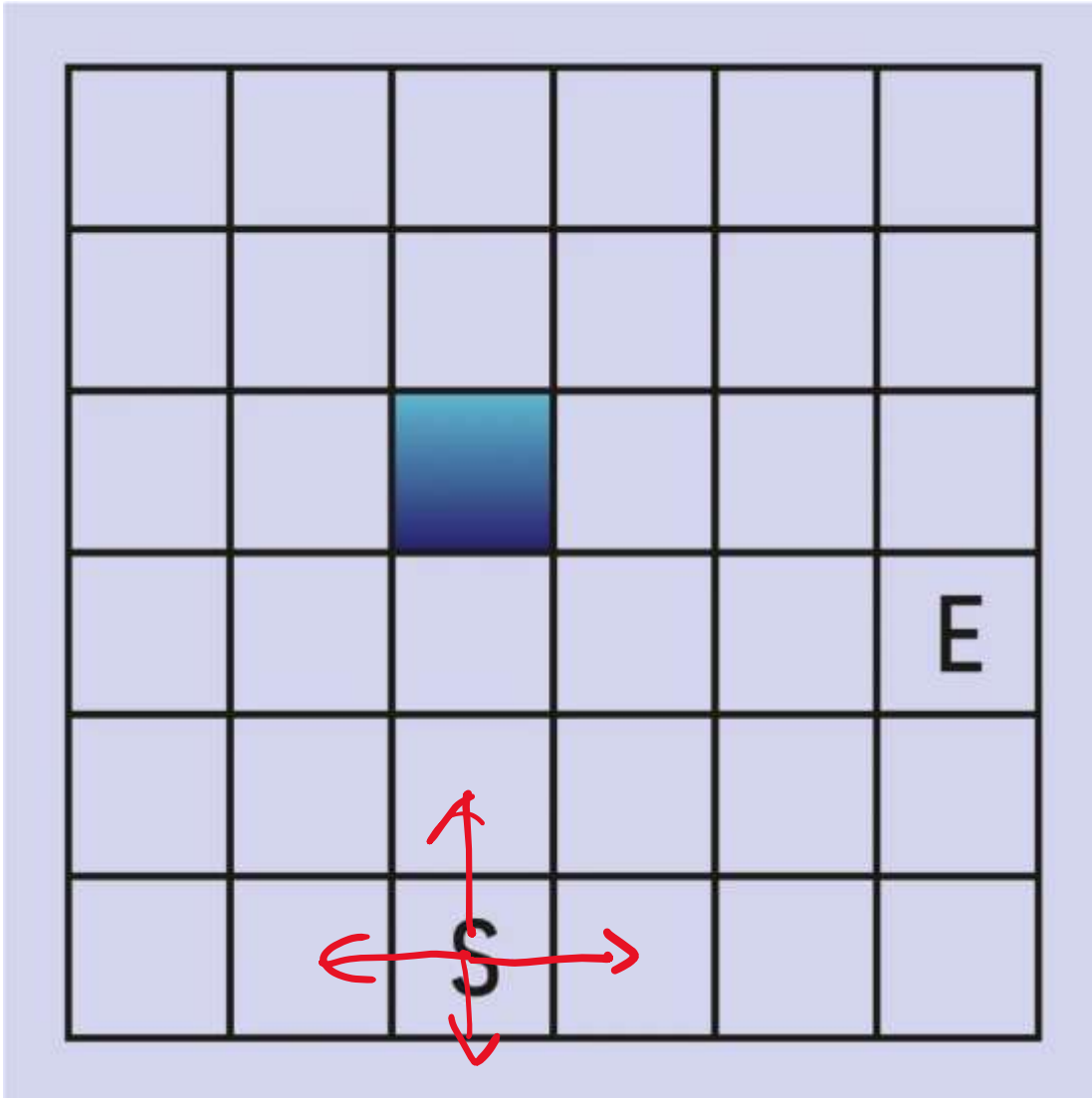
- **Best-First**
- **A***

<https://medium.com/omarelgabrys-blog/path-finding-algorithms-f65a8902eb40>

Demo: <http://qiao.github.io/PathFinding.js/visual/>

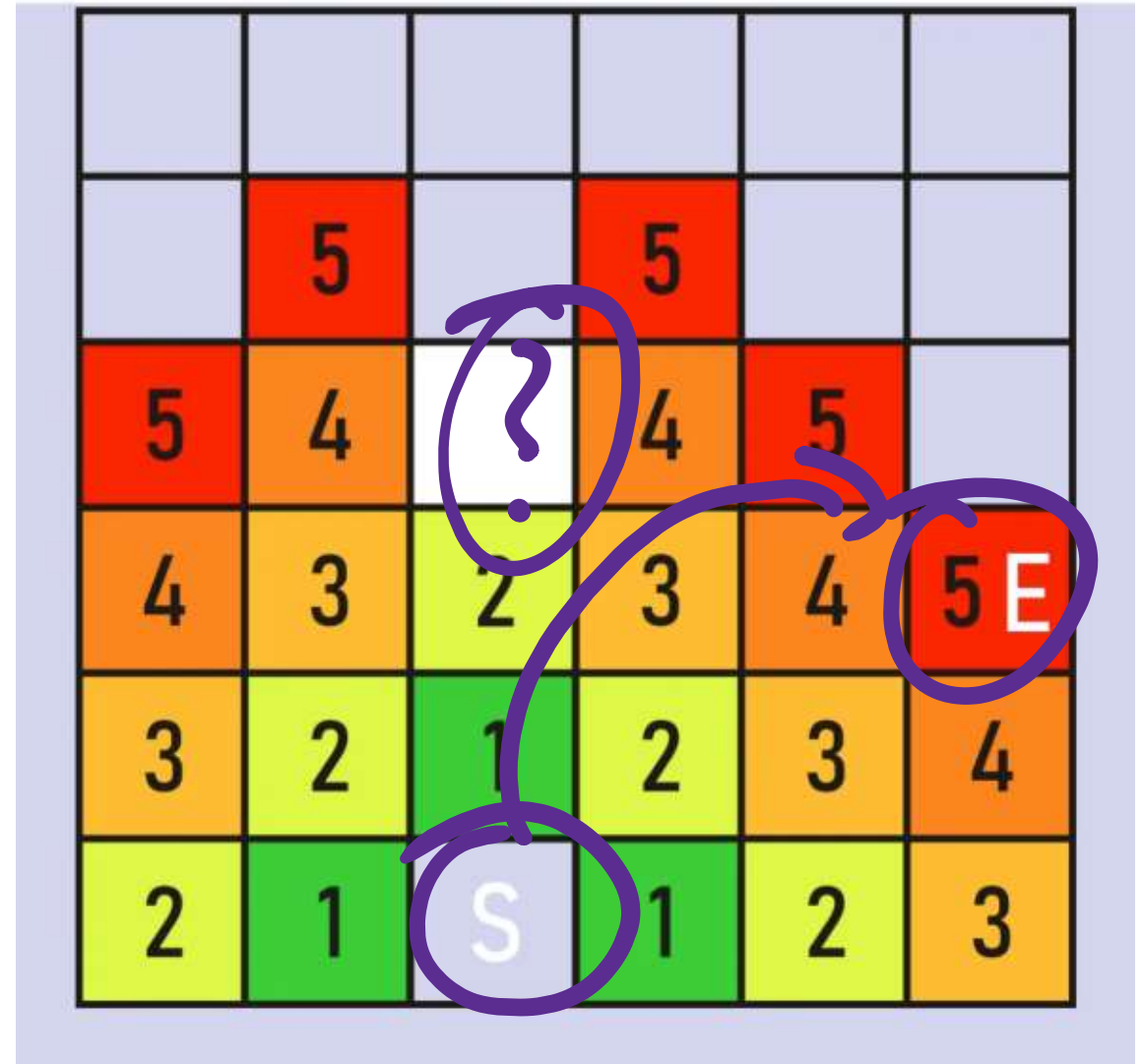
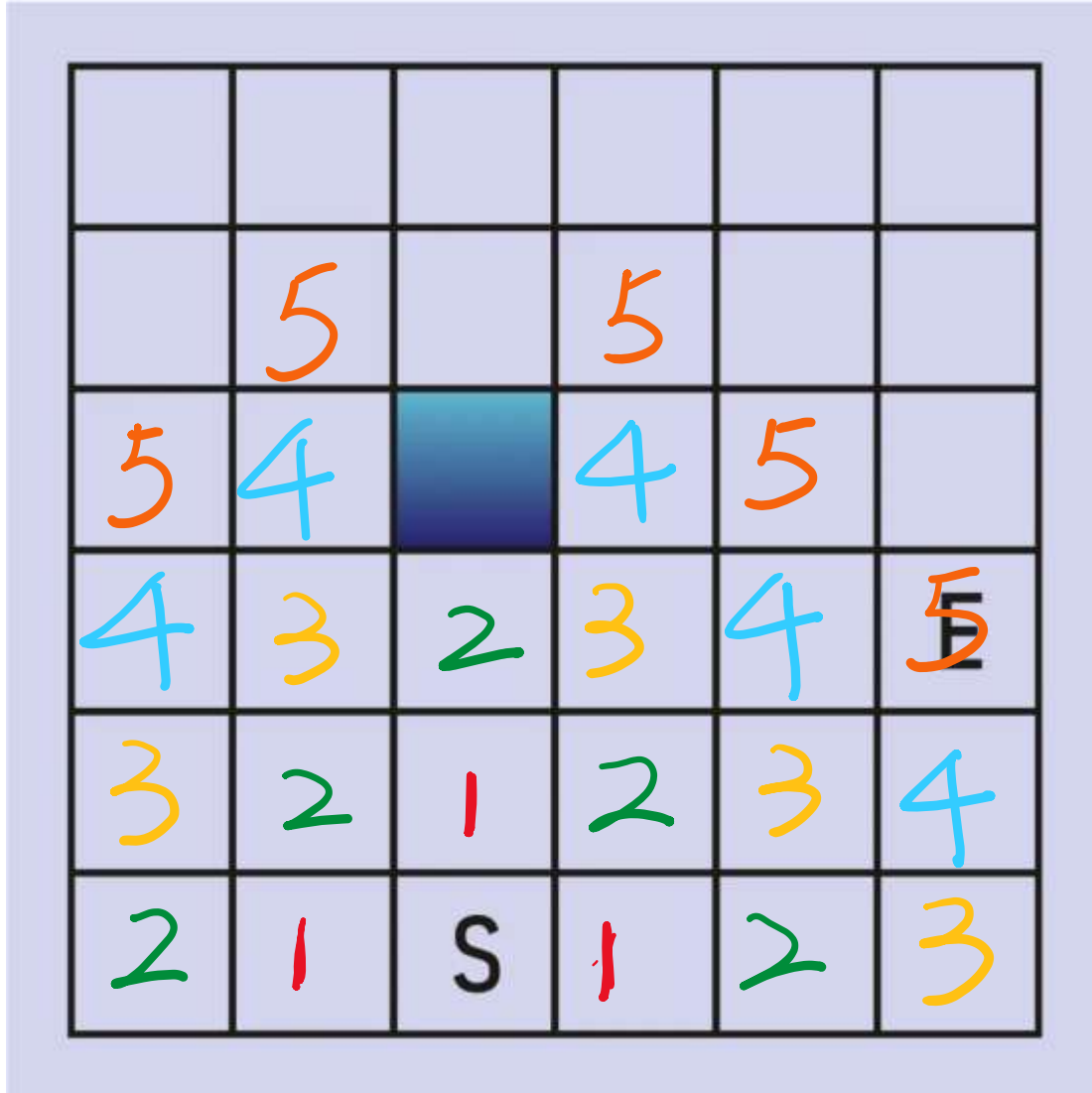


Breadth-First Search



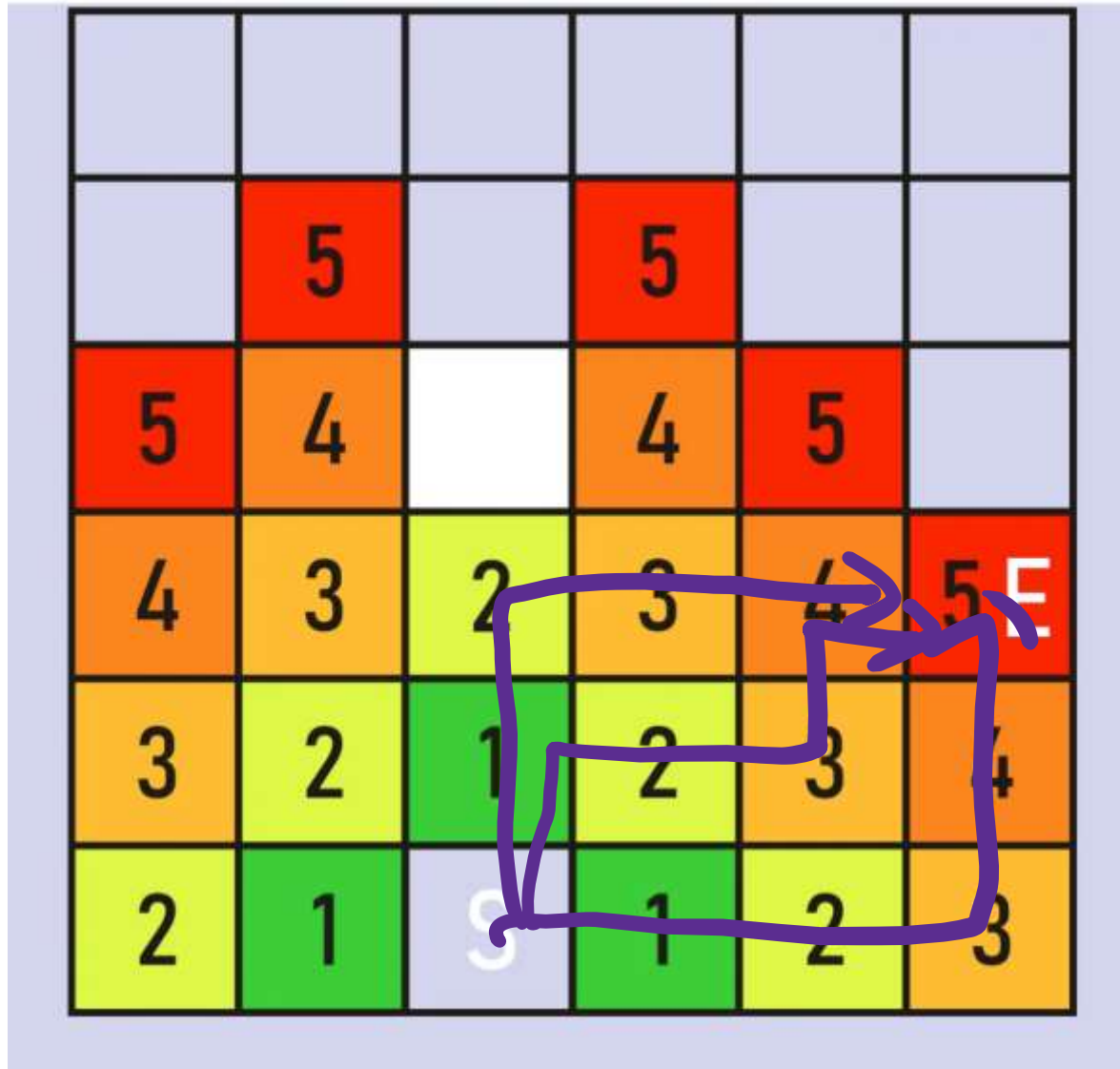


Breadth-First Search





Breadth-First Search



7



Breadth-First Search

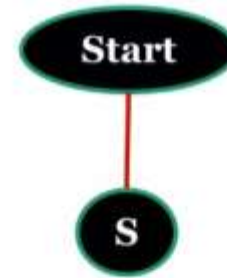
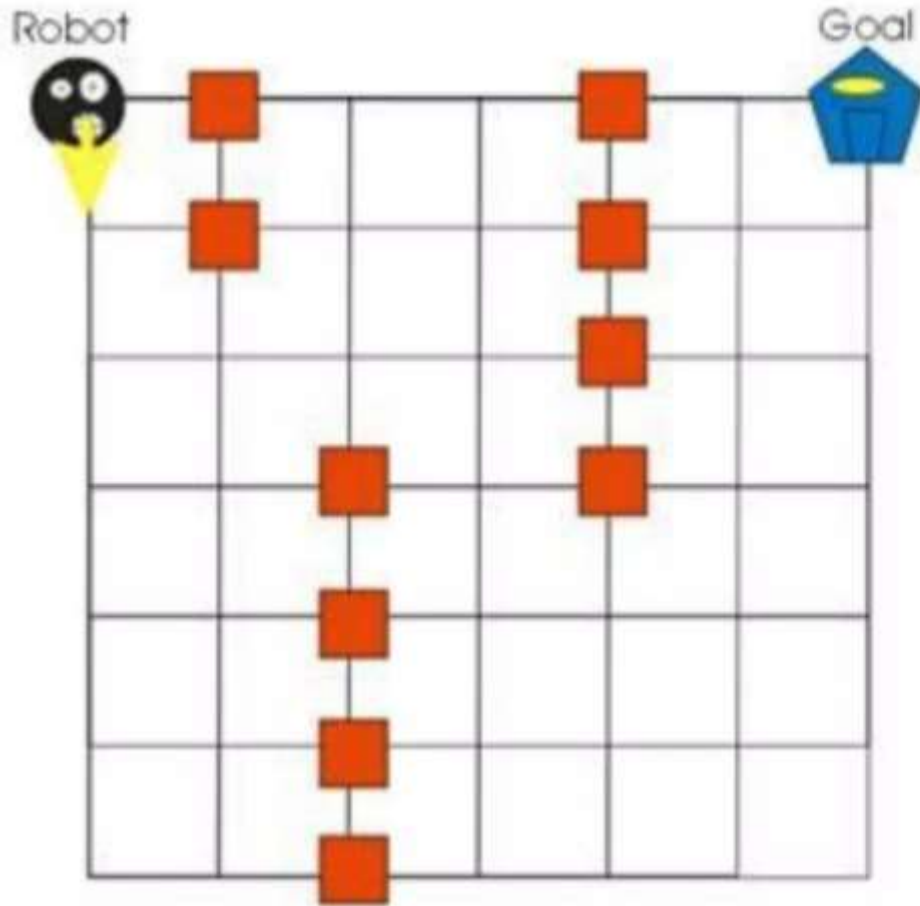
```
BFS(graph, start_node, end_node):  
    frontier = new Queue()  
    frontier.enqueue(start_node)  
    explored = new Set()  
  
    while frontier is not empty:  
        current_node = frontier.dequeue()  
        if current_node in explored: continue  
        if current_node == end_node: return success  
  
        for neighbor in graph.get_neighbors(current_node):  
            frontier.enqueue(neighbor)  
  
    explored.add(current_node)
```

| | | | | | |
|---|---|---|---|---|----|
| | | | | | |
| | 5 | | 5 | | |
| 5 | 4 | | 4 | 5 | |
| 4 | 3 | 2 | 3 | 4 | 5E |
| 3 | 2 | 1 | 2 | 3 | 4 |
| 2 | 1 | S | 1 | 2 | 3 |



Breadth-First Search

- Breadth-first Search (BFS)



Queue



Start

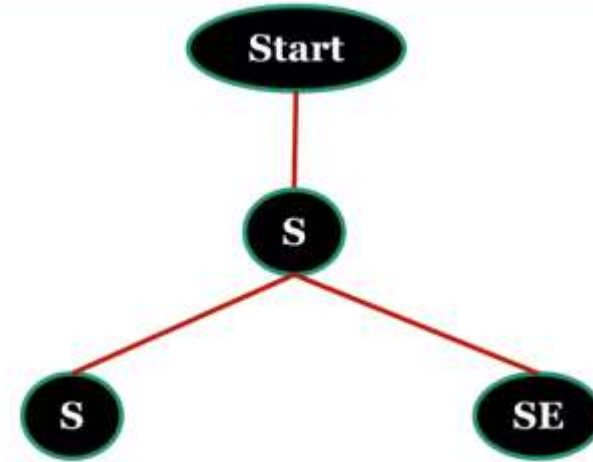
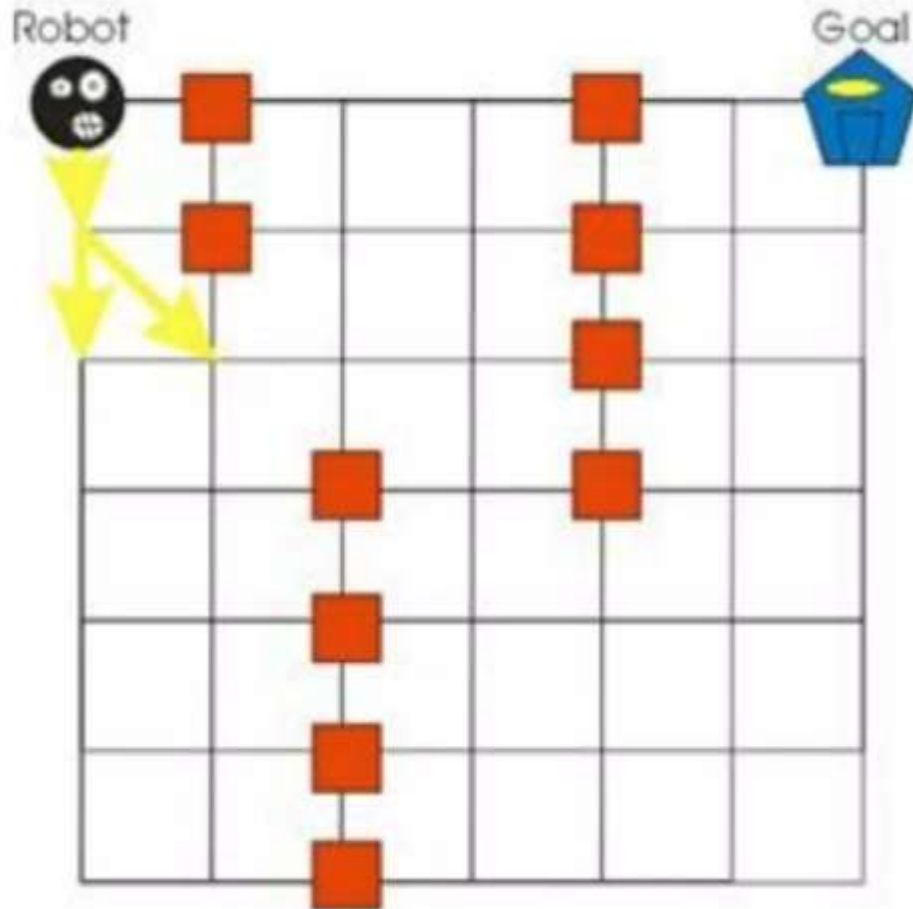
IN

OUT



Breadth-First Search

- **Breadth-first Search (BFS)**



Queue



Start S

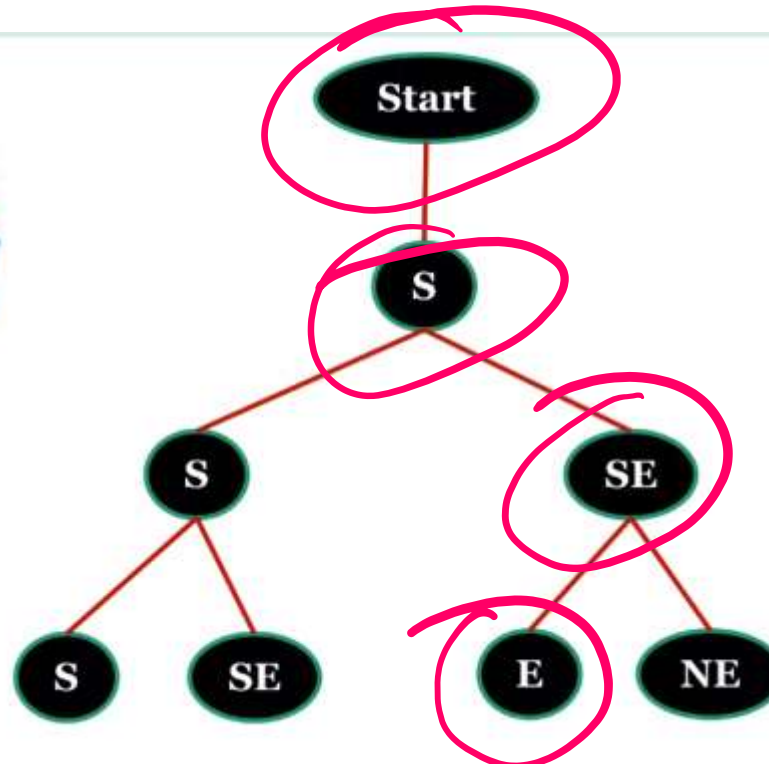
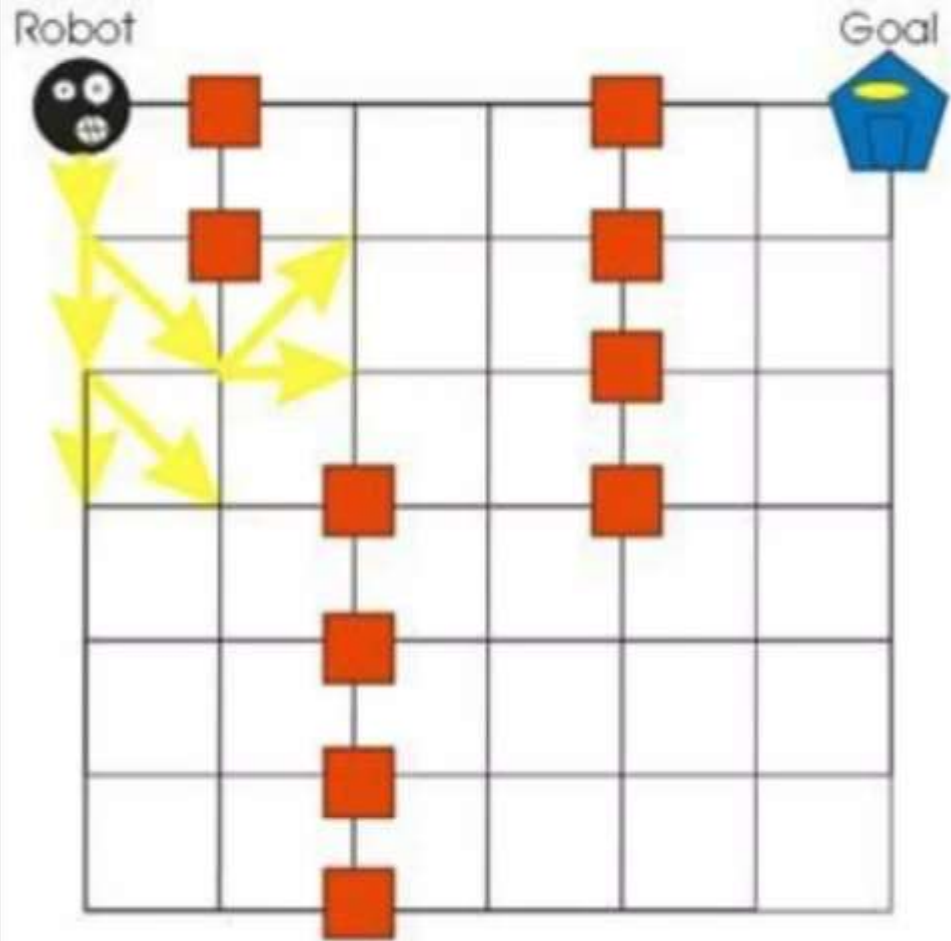
IN

OUT



Breadth-First Search

• Breadth-first Search (BFS)



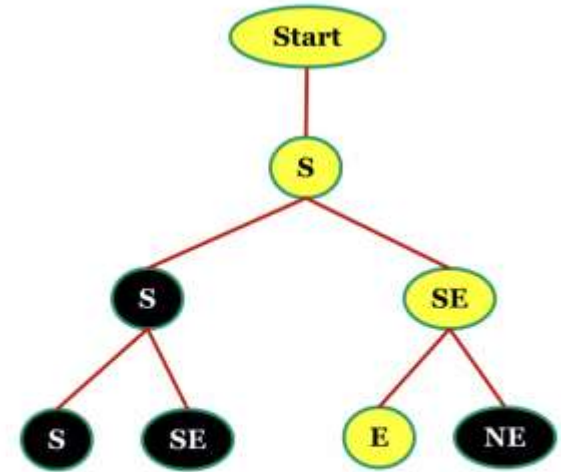
Queue



Start S S SE

IN

OUT



Assume that E is the goal,
Path is: **Start**→**S**→**SE**→**E**



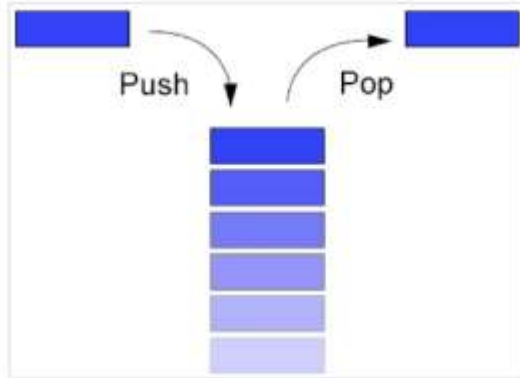
Breadth-First Search

Breadth-first Search (BFS)

- ◇ High memory requirement.
- ◇ **Exhaustive** search as it will process every node.
- ◇ Doesn't get stuck.
- ◇ Finds the shortest path (minimum number of steps).



Depth-First Search



Stack (LIFO)



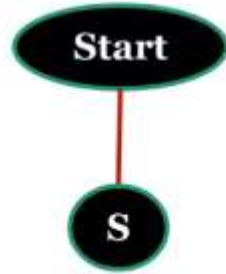
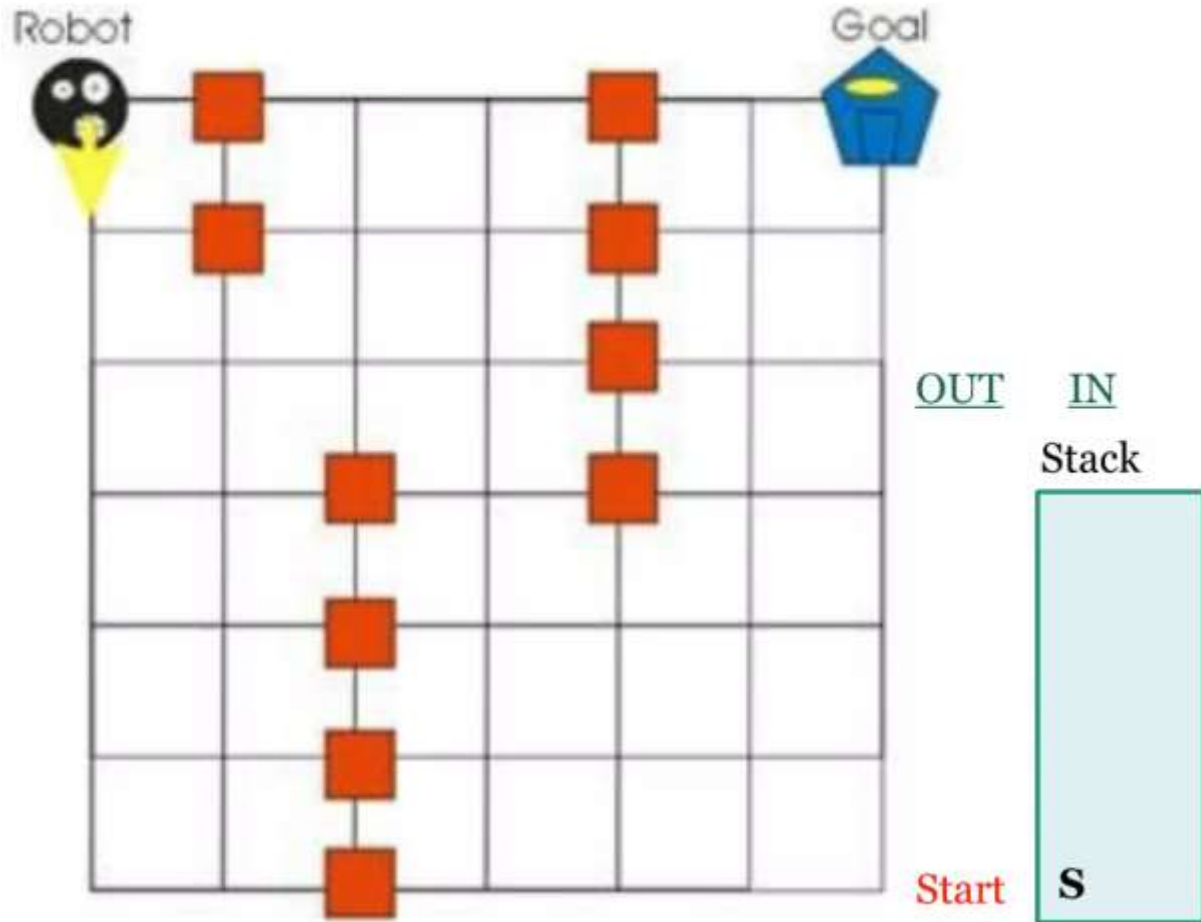
<http://wiki.ugrnet.jcs.in/t/Stack>

Depth first search is dead simple. First, go to the specified start node. Now, arbitrarily pick one of that node's neighbors and go there. If that node has neighbors, arbitrarily pick one of those and go there *unless we've already seen that node*. And we just repeat this process until one of two things happens. If reach the specified end node we terminate the algorithm and report success. If we reach a node with only neighbors we've already seen, or no neighbors at all, we go back one step and try one of the neighbors we *didn't* try last time.



Depth-First Search

• Depth-first Search (DFS)



```
DFS(graph, start_node, end_node):
    frontier = new Stack()
    frontier.push(start_node)
    explored = new Set()

    while frontier is not empty:
        current_node = frontier.pop()
        if current_node in explored: continue
        if current_node == end_node: return success

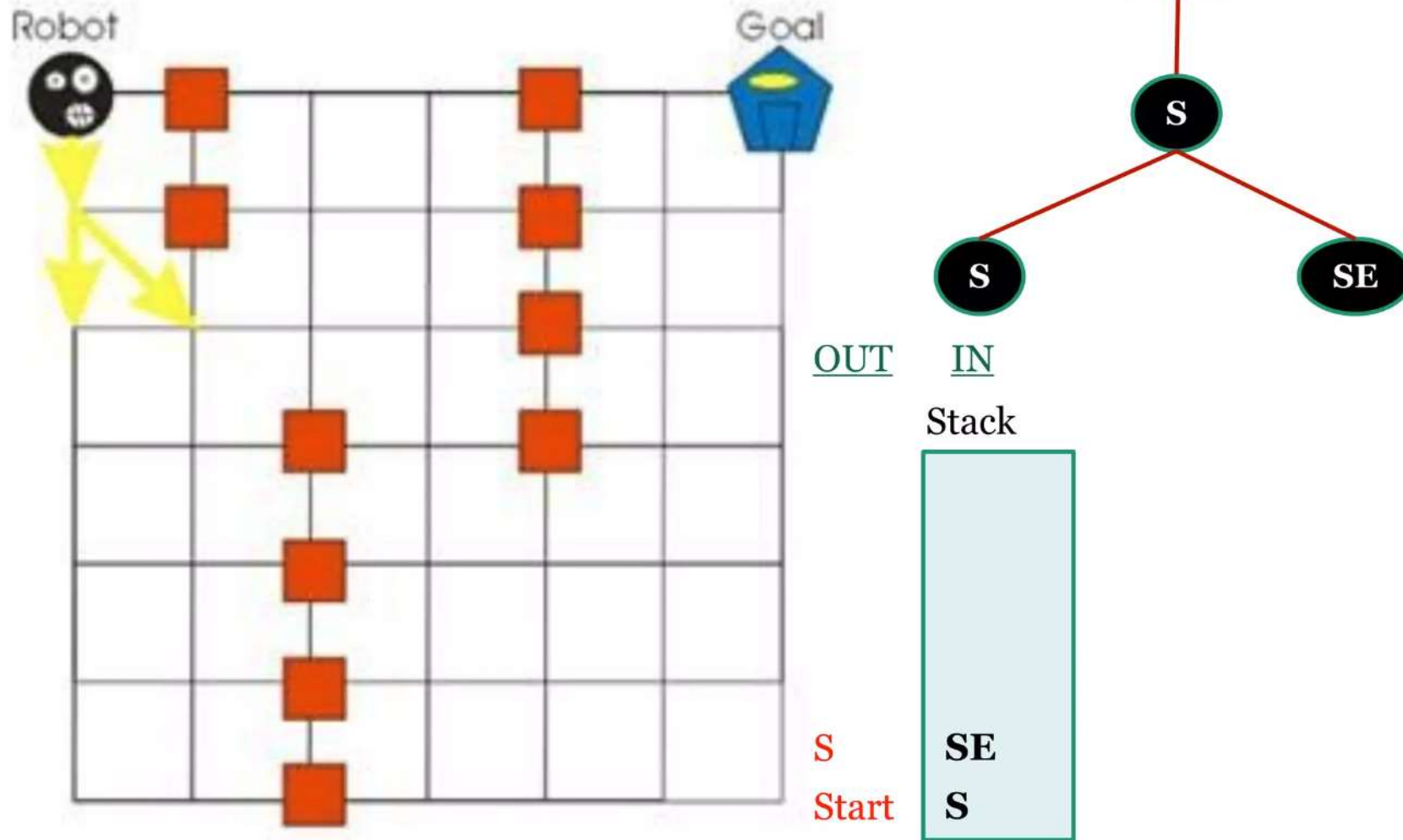
        for neighbor in graph.get_neighbors(current_node):
            frontier.push(neighbor)

        explored.add(current_node)
```



Depth-First Search

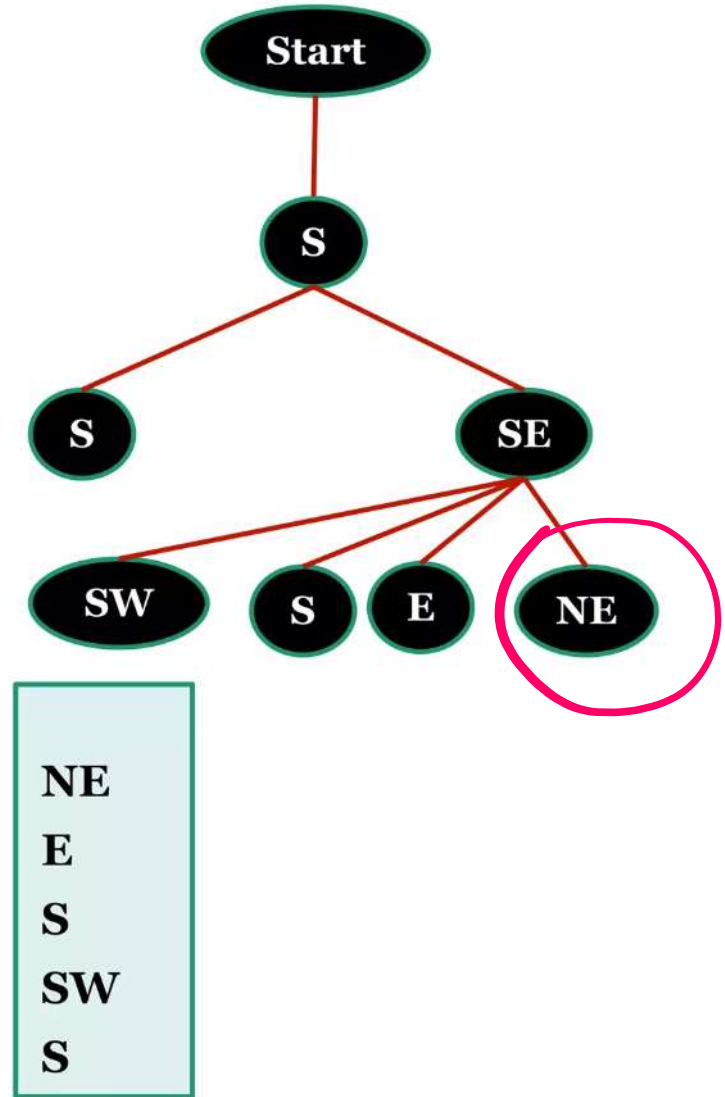
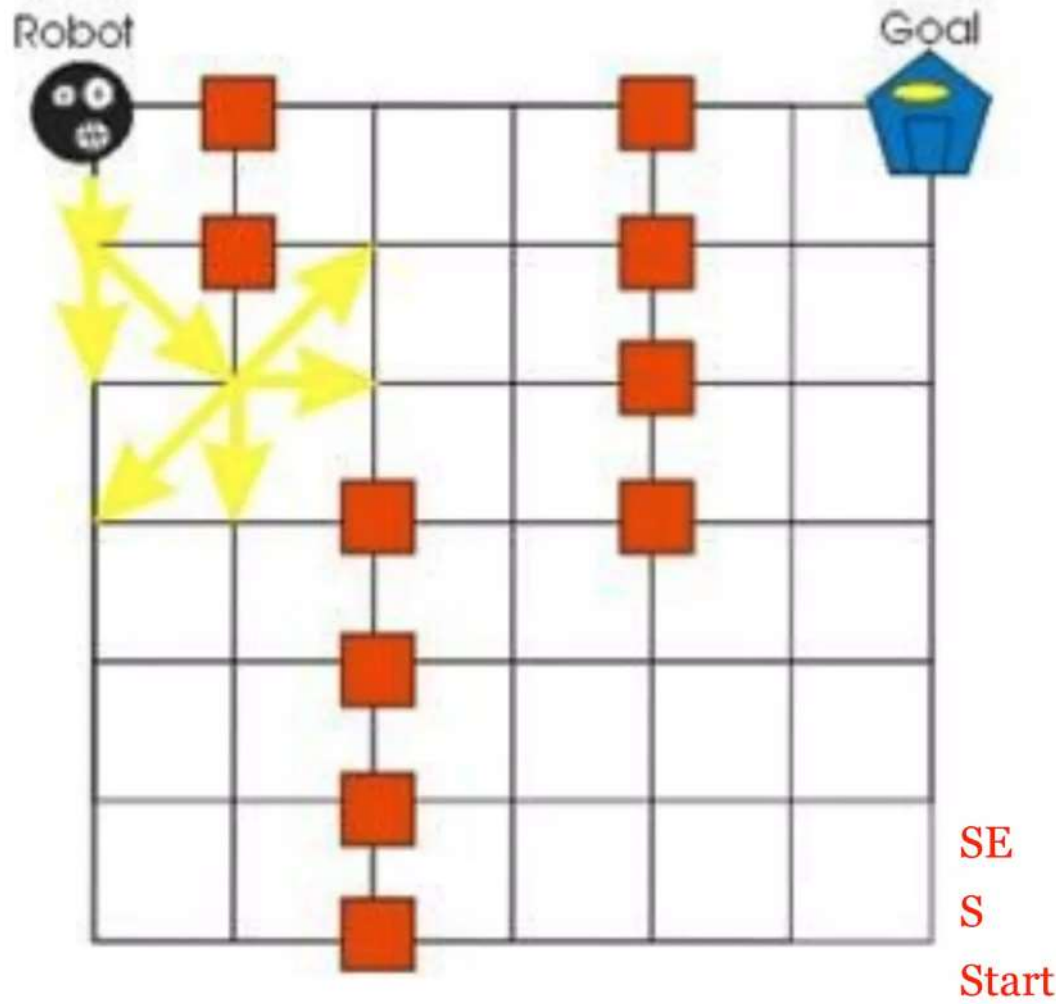
- **Depth-first Search (DFS)**





Depth-First Search

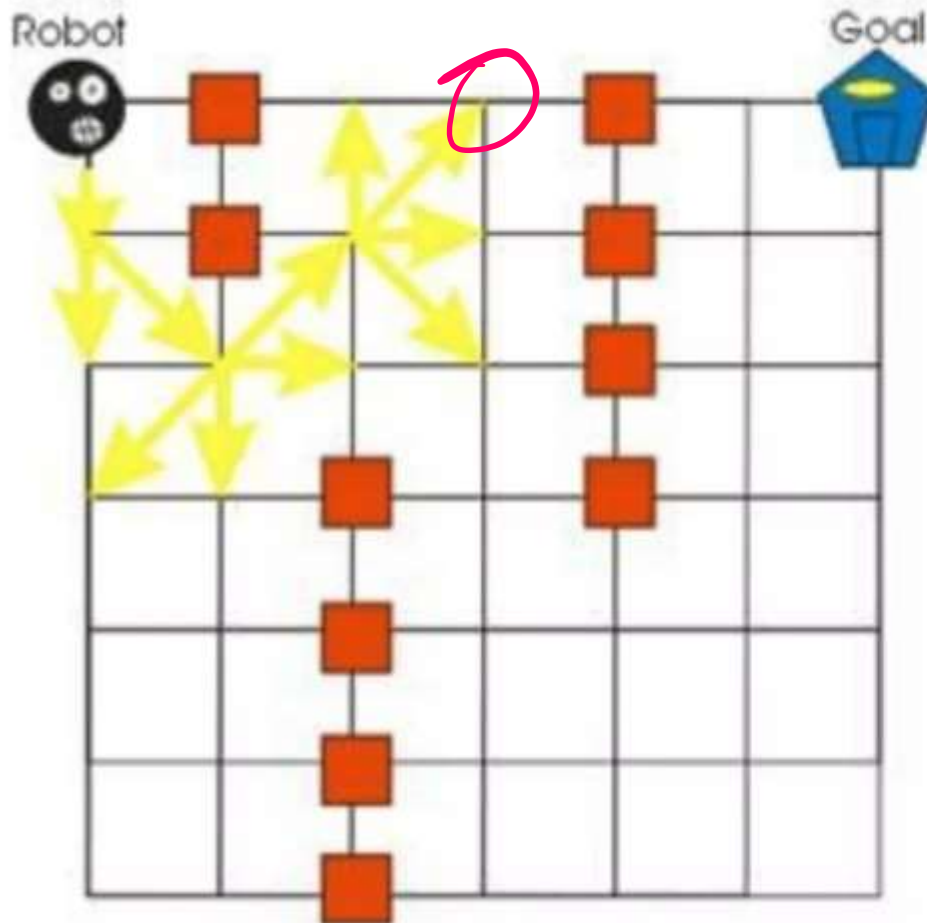
- Depth-first Search (DFS)





Depth-First Search

- **Depth-first Search (DFS)**



- ◇ The next node to be expanded would be **NE** and its successors would be added to the stack and this loop continues until the goal is found.
- ◇ Once the goal is found, you can then **trace back** through the tree to obtain the path for the robot to follow.



Brute-Force Search



ChatGPT • Bot

What is brute-force search?

ChatGPT

Brute-force search is a method of solving a problem by trying every possible solution until the right one is found. It is also called "exhaustive search" or "generate and test" method. This method is not always efficient, especially when the problem has a large solution space. But it is guaranteed to find the correct solution if one exists.

Tell me more



Can you give an example of a problem that can be solved using brute-force search?



What are the advantages and disadvantages of using brute-force search?



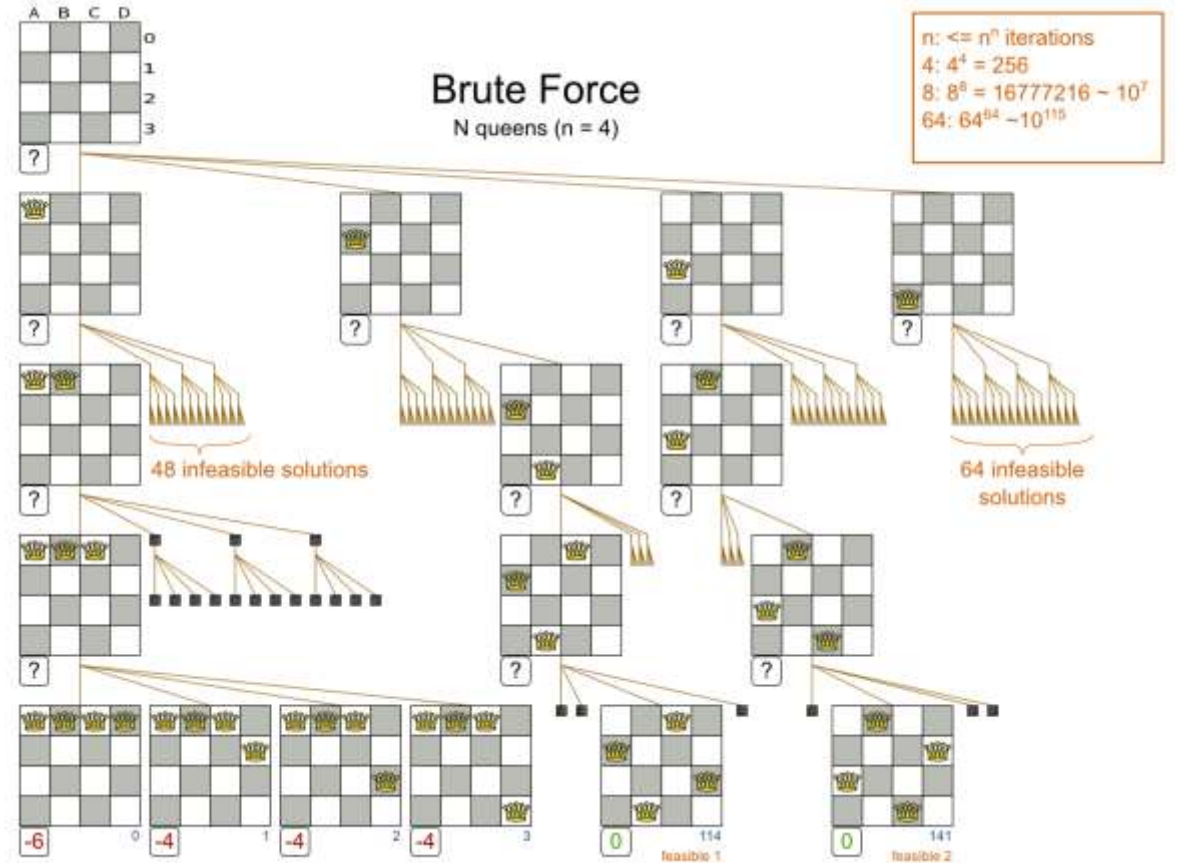
Are there any problems that cannot be solved using brute-force search?



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Discrete Planning

Blind.

- **Breadth-First Search**
- **Depth-First Search**
- **Brute-Force Search**

Informed

- **Best-First**
- **A***

Only use the starting nodes

goal?

<https://medium.com/omarelgabrys-blog/path-finding-algorithms-f65a8902eb40>

<http://qiao.github.io/PathFinding.js/visual/>



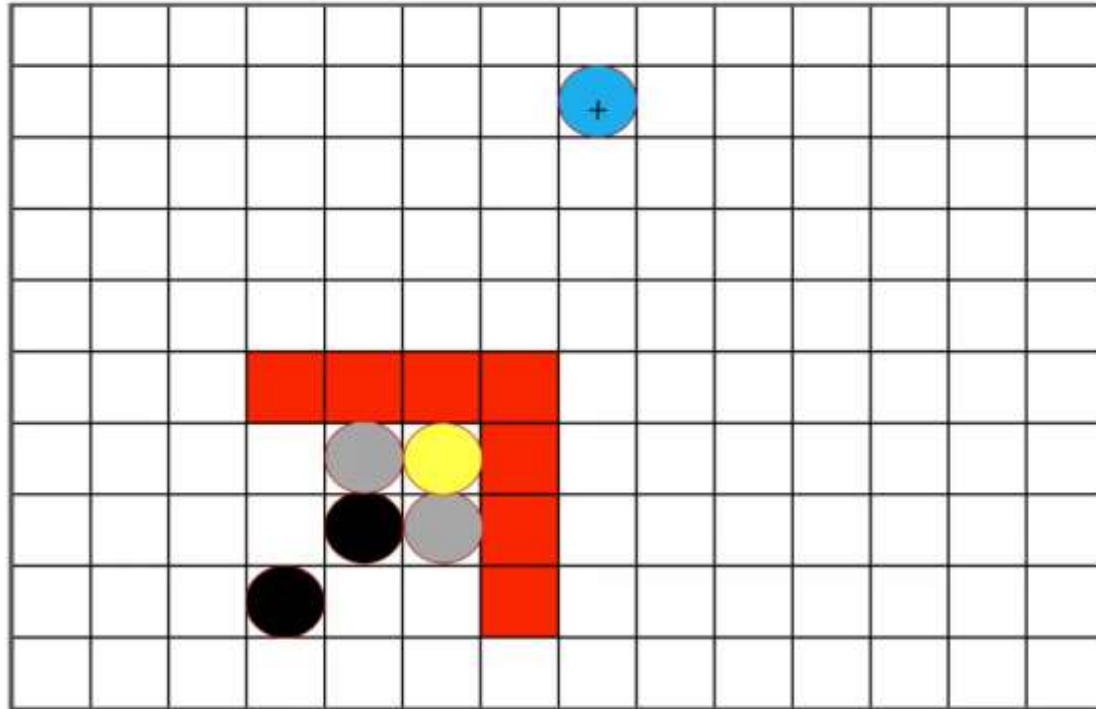
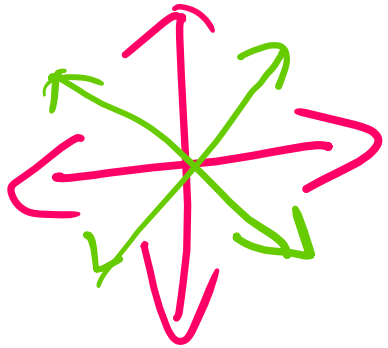
Best-First Search

- **Best-first**

1. **Workspace** discretized into cells
2. **Insert** (x_{init}, y_{init}) into list **OPEN**
3. **Find** all **8-way neighbors** to (x_{init}, y_{init}) that have not been previously visited and insert into OPEN
4. **Sort** neighbors by minimum potential
5. **Form** paths from neighbors to (x_{init}, y_{init})
6. **Delete** (x_{init}, y_{init}) from OPEN
7. $(x_{init}, y_{init}) = \text{minPotential}(\text{OPEN})$
8. **GOTO** 2 until $(x_{init}, y_{init}) = \text{goal}$ (SUCCESS) or OPEN empty (FAILURE)



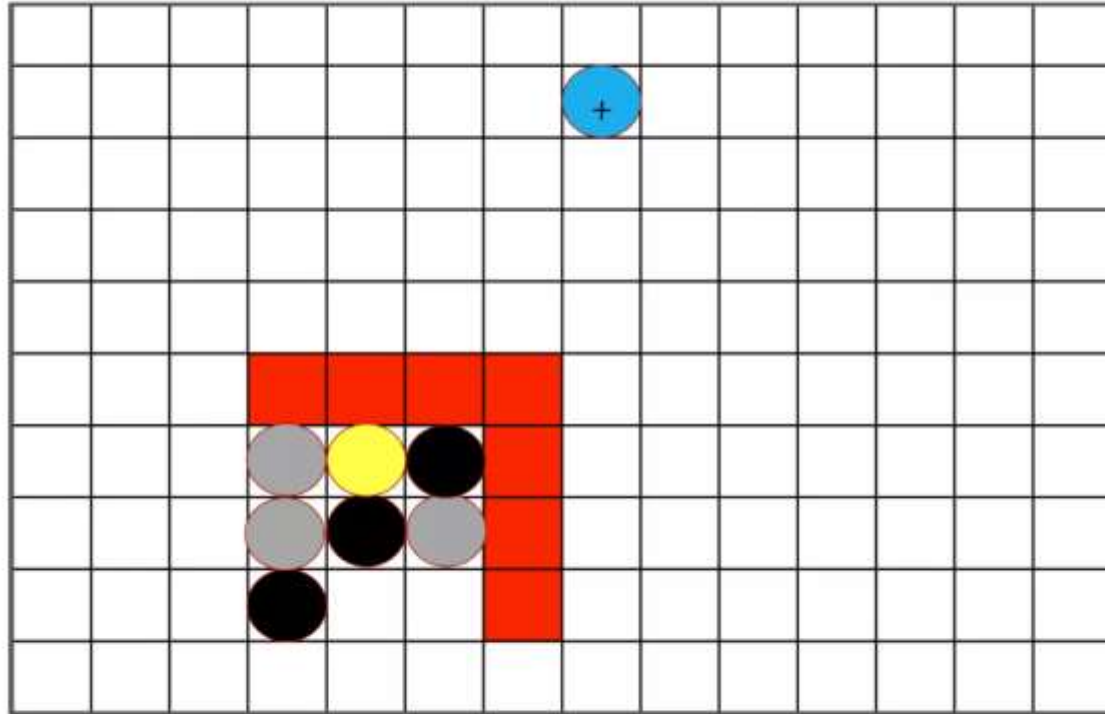
Best-First Search



- Goal
- Neighbor
- Visited
- Local minimum detected
- Best step
- Obstacle



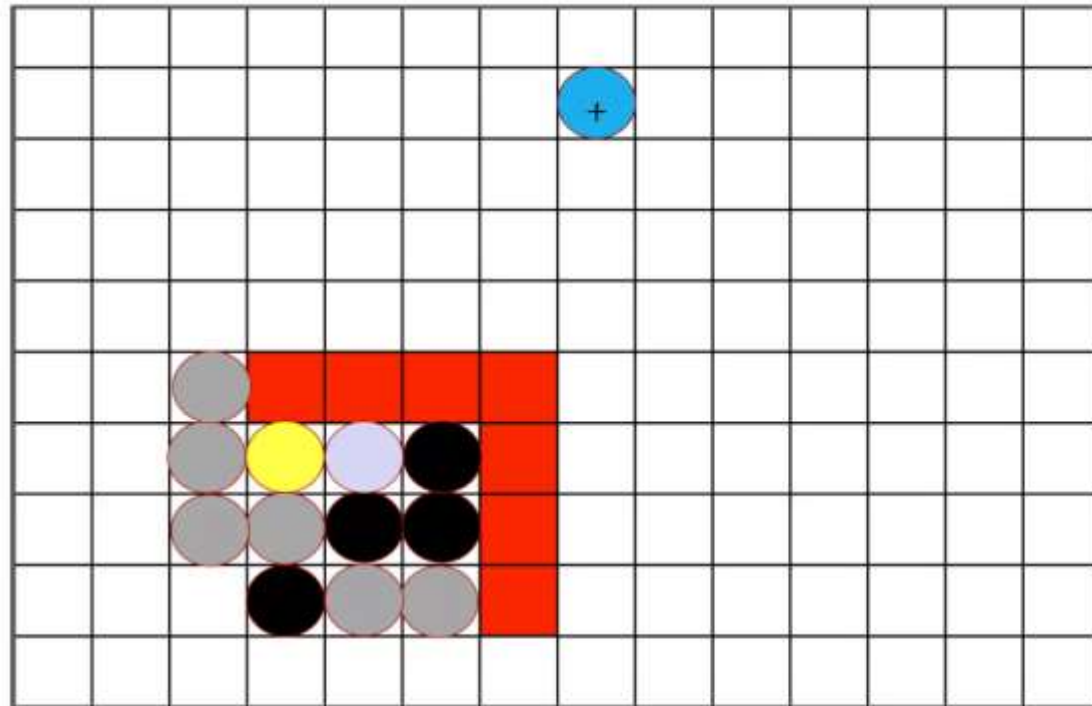
Best-First Search



- Goal
- Neighbor
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- Local minimum detected
- Best step
- Obstacle



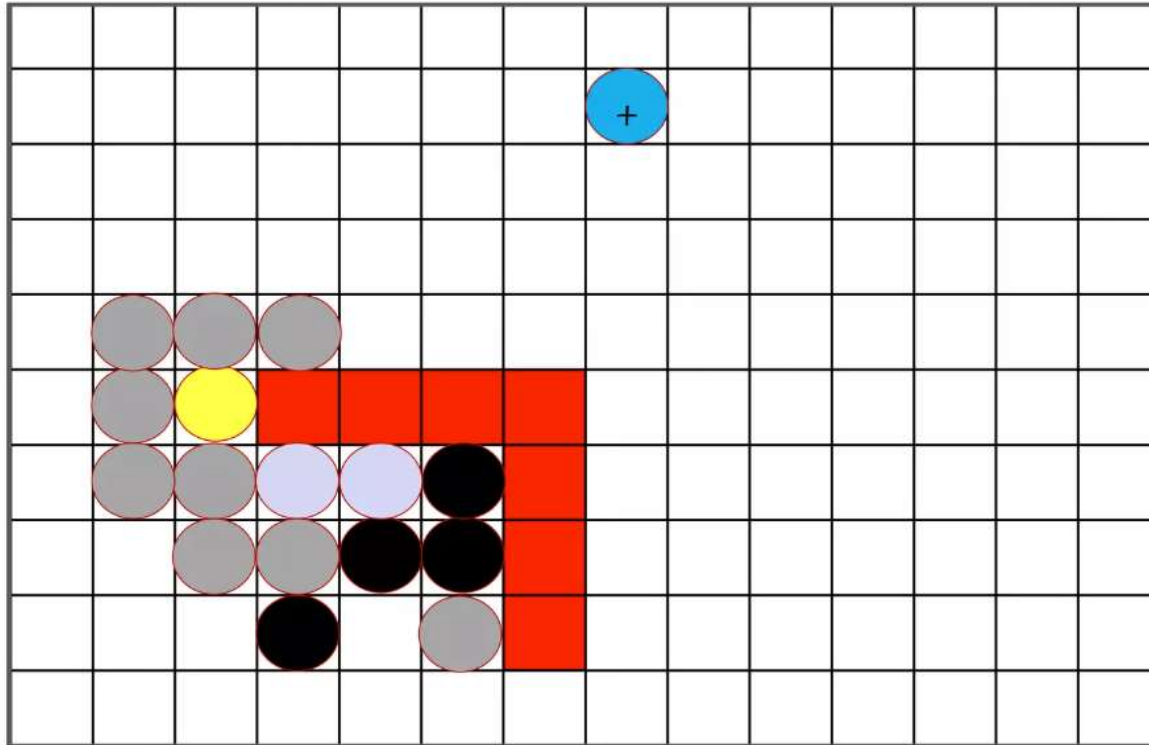
Best-First Search



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- Neighbor
- Visited
- Local minimum detected
- Best step
- Obstacle



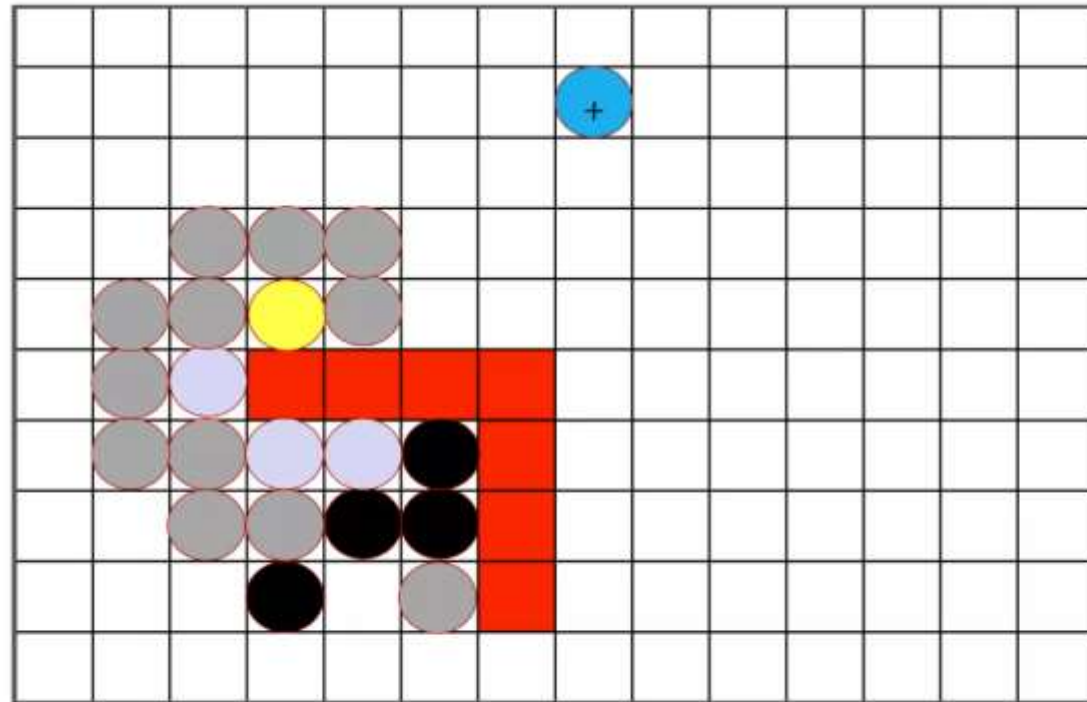
Best-First Search



- Goal
- Neighbor
- Visited
- Local minimum detected
- Best step
- Obstacle



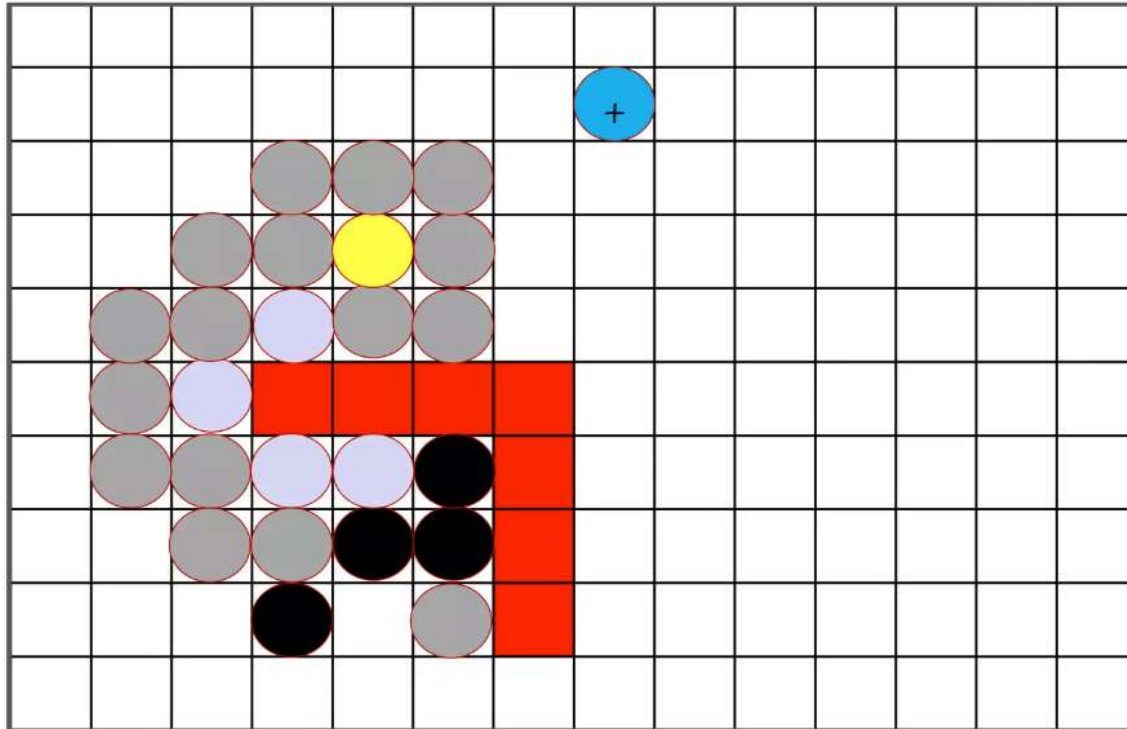
Best-First Search



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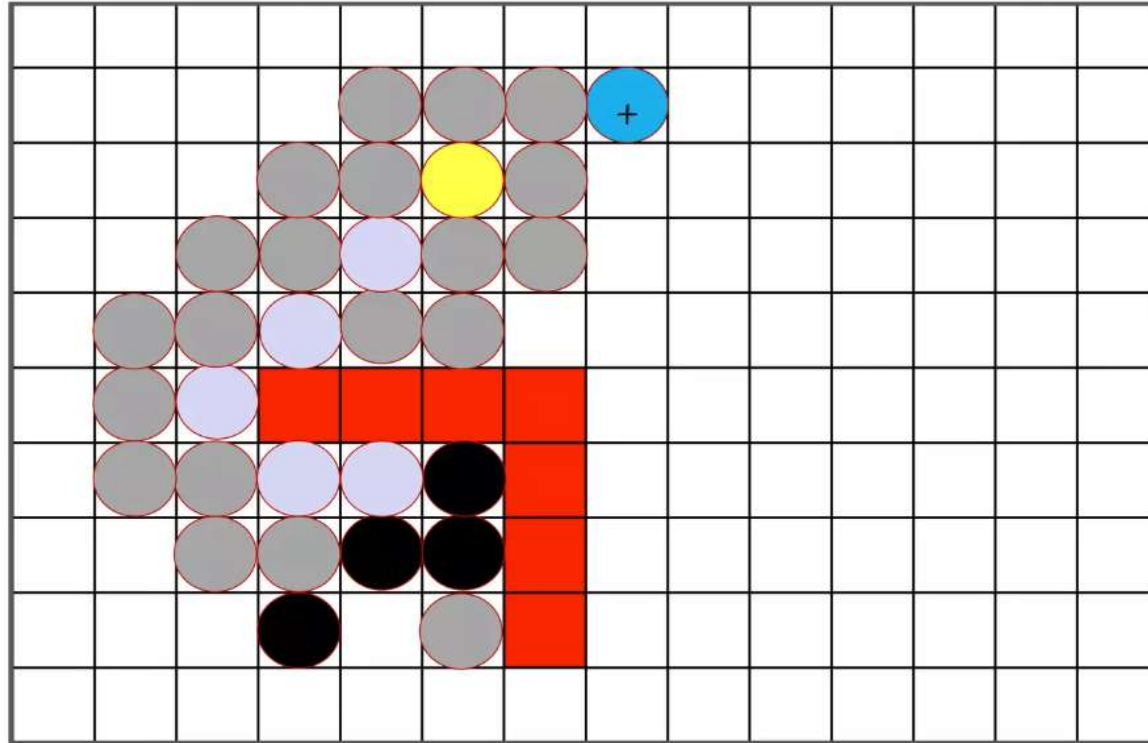
Best-First Search



- Goal
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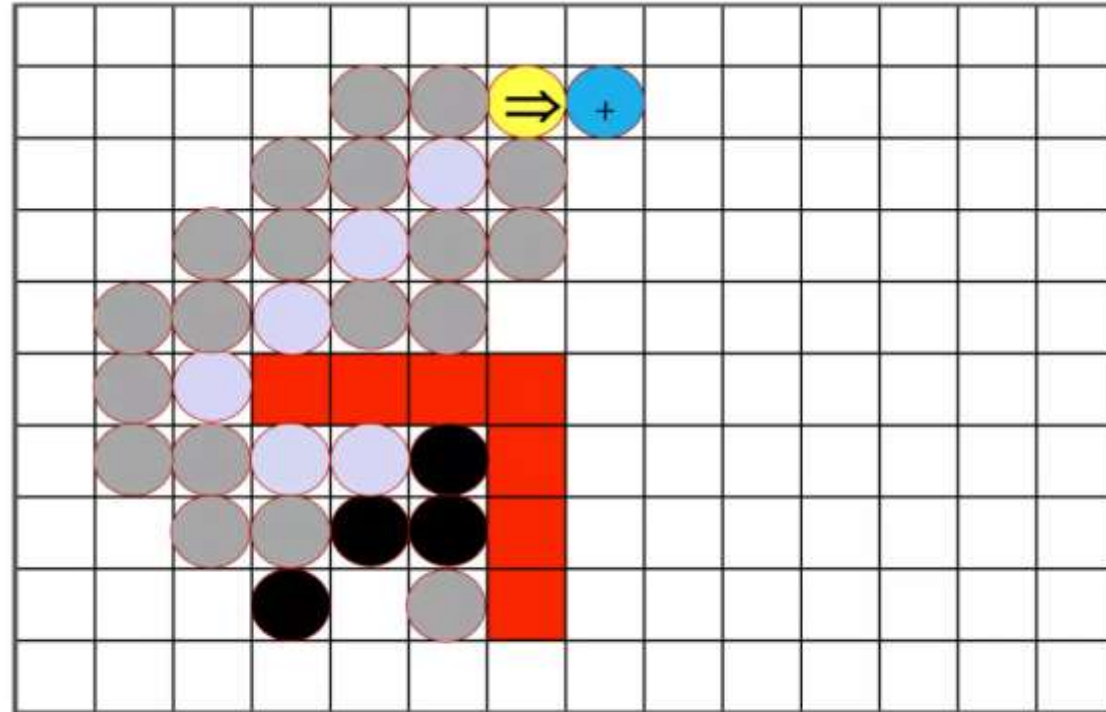
Best-First Search



- Goal
- Neighbor
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- Local minimum detected
- Best step
- Obstacle



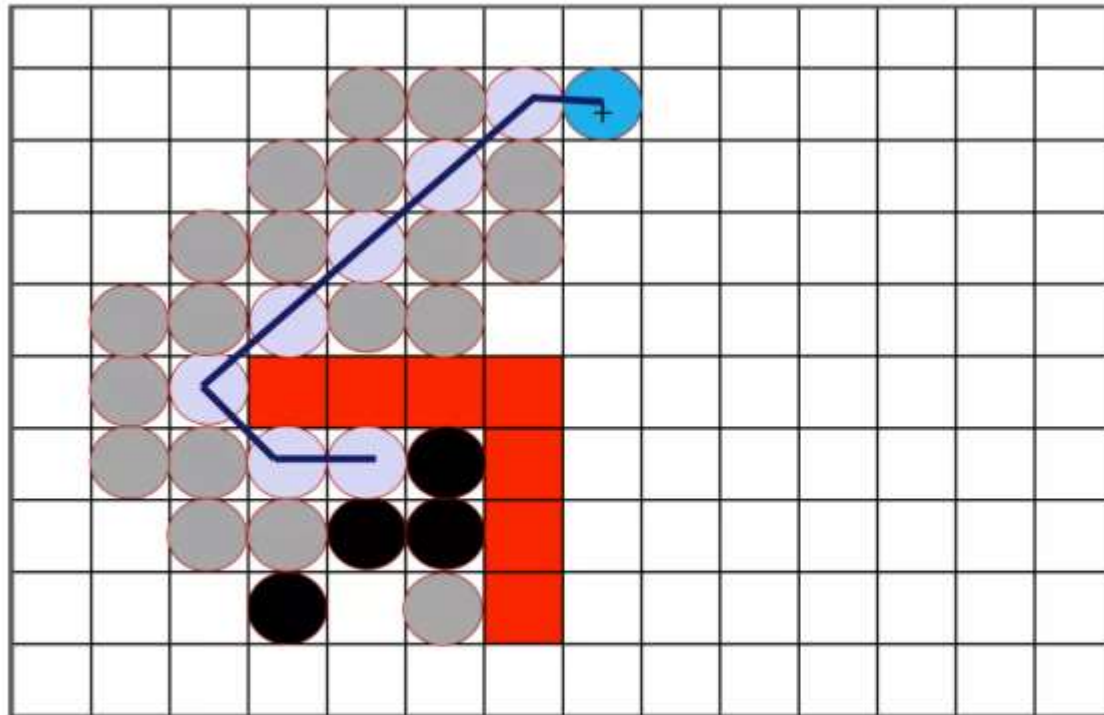
Best-First Search



- Goal
- Neighbor
- Visited
- Local minimum detected
- Best step
- Obstacle



Best-First Search



- Goal
- Neighbor
- Visited
- Local minimum detected
- Best step
- Obstacle



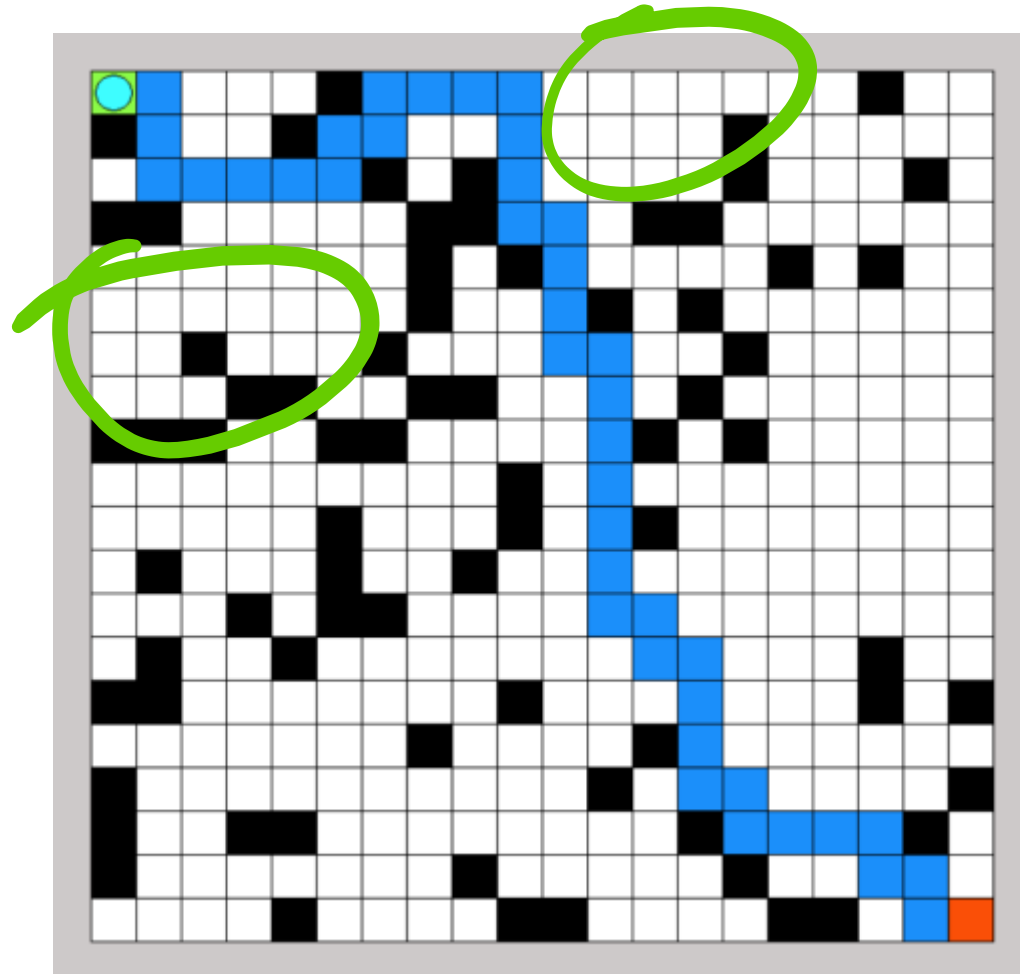
Best-First Search

- **Best-first**

- ◇ It is a kind of **mixed depth and breadth first** search.
- ◇ Adds the successors of a node to the expand list.
- ◇ All nodes on the list are sorted according to the **heuristic values**.
- ◇ Expand **most desirable unexpanded** node.
- ◇ Special Case: **A***.



A^*



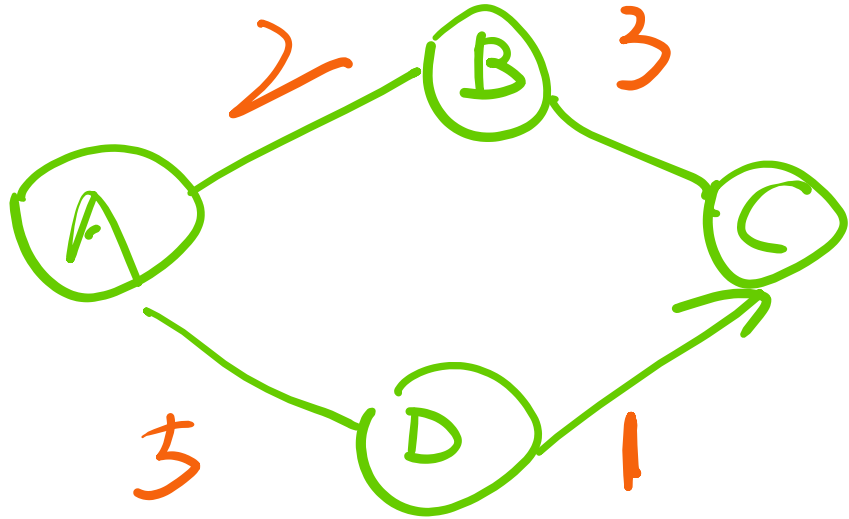
https://en.wikipedia.org/wiki/A*_search_algorithm

<https://www.redblobgames.com/pathfinding/a-star/introduction.html>



A*

Start Node.



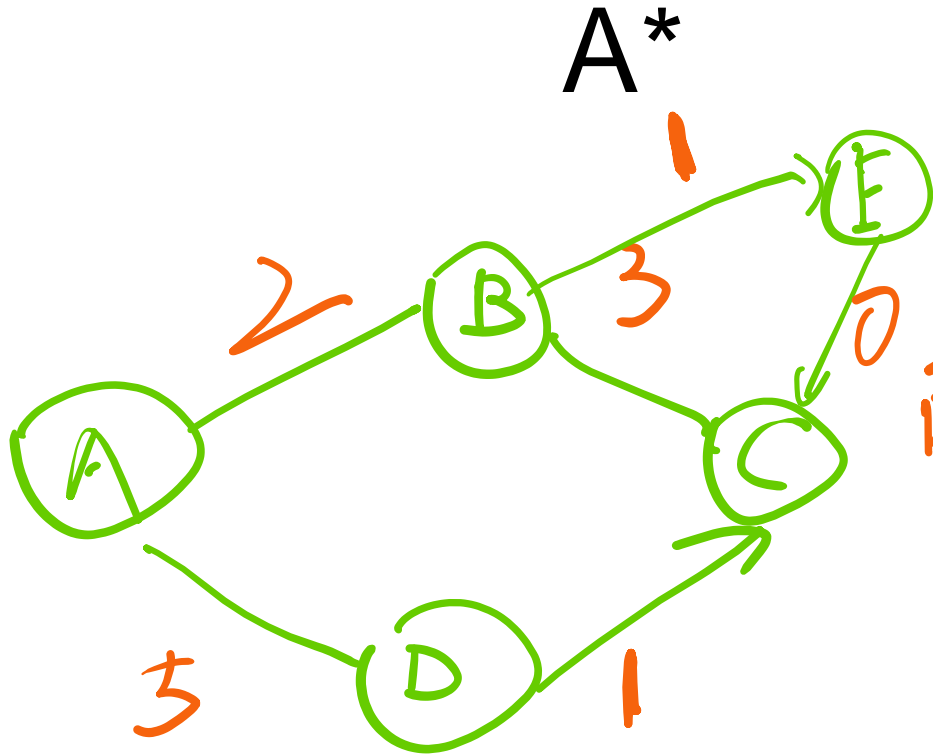
End Node

$$A-B-C = 2+3 = \underline{5} \quad \checkmark$$

$$A-D-C = 5+1 = \underline{6}$$



Start Node.



End Node

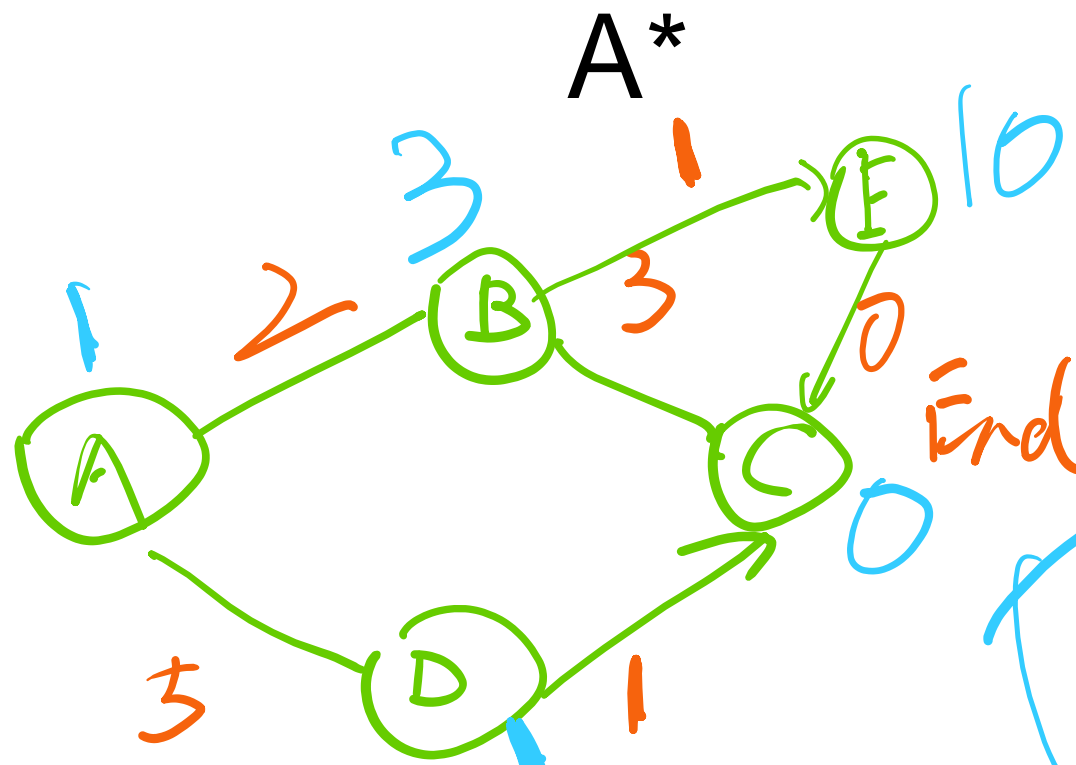
$$A-B-C = 2+3 = \underline{5}$$

$$A-D-C = 5+1 = 6$$

$$A-B-E-C = 2+1 = \underline{3} \checkmark$$



Start Node.



End Node

Heuristic value.

path weight $g(n)$

$h(n)$

★ cost function $f(n) = g(n) + h(n)$



A^*

$$f(n) = g(n) + h(n)$$

↓ ↓ ↓
total cost dist(start) dist(goal)



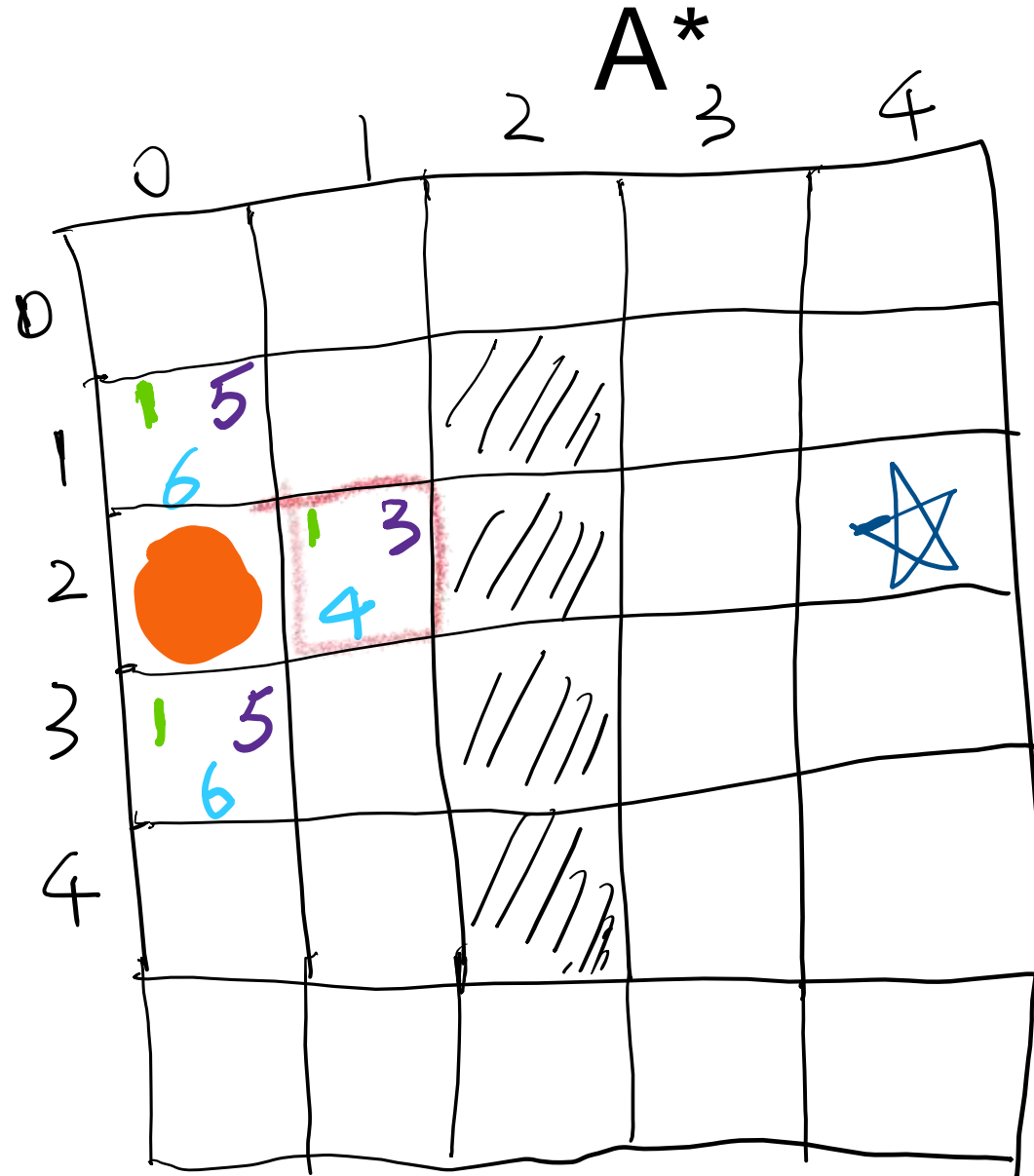
estimated distance from the current node to the end node



$h(n)$

Manhattan Distance

$$d_{12} = |x_1 - x_2| + |y_1 - y_2|$$



$$f(n) = g(n) + h(n)$$

open-list (to visit)

(1, 2), p: start

(0, 1), p: start

(0, 3), p: start

closed-list:



A^*

| | 0 | 1 | 2 | 3 | 4 |
|---|--------|--------|--------|--------|------|
| 0 | | | | | |
| 1 | 1 6 | 5 6 | 2 1 | 4 3 | //// |
| 2 | | | | | //// |
| 3 | 1 6 | 5 6 | 2 6 | 4 6 | //// |
| 4 | | | | | //// |

$$f(n) = g(n) + h(n)$$

open-list (to visit)

(1, 1) p: (1, 2) 6
(1, 3) p: (1, 2), 6

closed-list:
(1, 2), p: Start.



A^*

| | | | | | |
|---|----------|----------|------|---|---|
| | 0 | 1 | 2 | 3 | 4 |
| 0 | | | | | |
| 1 | 1 5 6 | 2 4 5 | //// | | |
| 2 | ● | 1 3 4 | //// | | ☆ |
| 3 | 1 5 6 | 2 4 5 | //// | | |
| 4 | | 3 5 8 | //// | | |

8 > 6

$$f(n) = g(n) + h(n)$$

open-list (to visit)

(1, 1) p: (1, 2) 6

(1, 4) p: (1, 2), 8 (?)

closed-list:

(1, 2), p: Start.



A^*

| | 0 | 1 | 2 | 3 | 4 |
|---|--------|---------------|----------|---|---|
| 0 | | | | | |
| 1 | 1 6 | 5 2 4 6 | /// | | |
| 2 | ● | 1 4 | 3 /// | | ☆ |
| 3 | 1 6 | 5 2 4 6 | /// | | |
| 4 | | 3 5 8 | /// | | |

$$f(n) = g(n) + h(n)$$

open-list (to visit)

$(1, 1)$ $p = (1, 2)$ 6

$(1, 4)$ $p = (1, 2)$ 8 (?)

closed-list:

$(1, 2)$, $p = \text{Start}$

$(1, 3)$ $p = (1, 2)$, 6



A^*

| | | | | | |
|---|----------|----------|----------|------------|---|
| | 0 | 1 | 2 | 3 | 4 |
| 0 | | 3 5 8 | 4 4 8 | 5 3 8 | |
| 1 | 1 5 6 | 2 4 6 | //// | 6 2 8 | |
| 2 | | 1 3 4 | //// | 7 1 8 → | |
| 3 | 1 5 6 | 2 4 6 | //// | | |
| 4 | | 3 5 8 | //// | | |

$$f(n) = g(n) + h(n)$$

open-list (to visit)

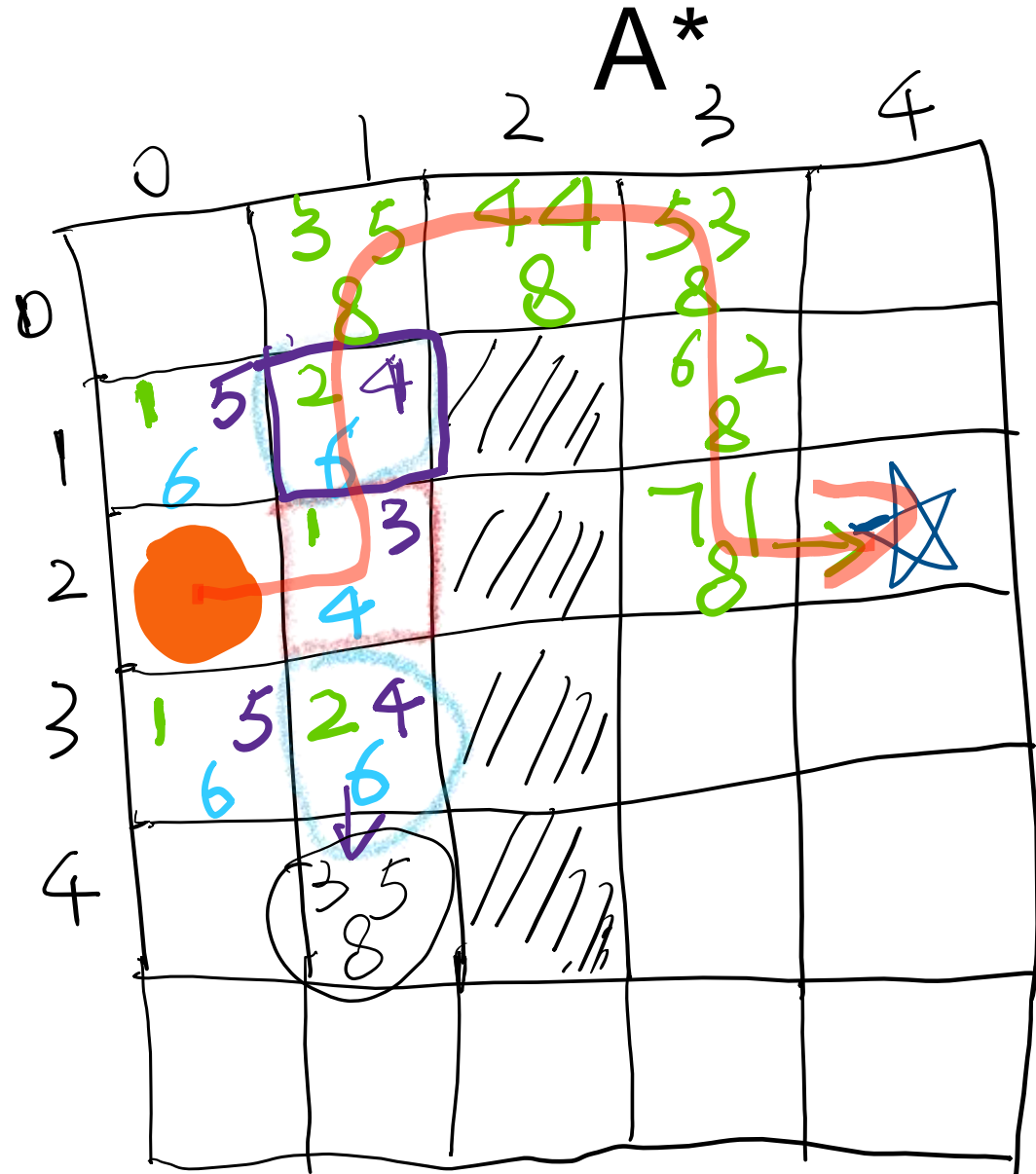
$(1, 1)$ $p = (1, 2)$ 6

$(1, 4)$ $p = (1, 2)$ 8 (?)

closed-list:

$(1, 2)$, $p = \text{Start}$

$(1, 3)$ $p = (1, 2)$, 6



$$f(n) = g(n) + h(n)$$

open-list (to visit)

(1, 1) p: (1, 2) 6
 (1, 4) p: (1, 2) 8

closed-list:

(1, 2), p: Start.
 (1, 3) p: (1, 2), 6



A* implementation

```
class Node:  
    def __init__(self, position, parent=None, cost=0):  
        self.position = position  
        self.parent = parent  
        self.cost = cost
```

```
def heuristic(node, goal):  
    x1, y1 = node.position  
    x2, y2 = goal.position  
    return abs(x1 - x2) + abs(y1 - y2)
```



```
def get_neighbors(node):  
    x, y = node.position  
    neighbors = []  
  
    # Add adjacent nodes (up, down, left, right)  
    for dx, dy in [(0, 1), (0, -1), (1, 0), (-1, 0)]:  
        new_x, new_y = x + dx, y + dy  
        neighbors.append(Node((new_x, new_y), parent=node, cost=node.cost + 1))  
  
    return neighbors
```

define neighbor nodes.



A* implementation

```
import heapq

def astar(start, goal):
    open_list = []
    closed_list = set()

    heapq.heappush(open_list, (start.cost, start))

    while open_list:
        current_cost, current_node = heapq.heappop(open_list)

        if current_node == goal:
            # Goal reached, construct and return the path
            path = []
            while current_node:
                path.append(current_node.position)
                current_node = current_node.parent
            return path[::-1]

        closed_list.add(current_node)

        for neighbor in get_neighbors(current_node):
            if neighbor in closed_list:
                continue

            new_cost = current_node.cost + 1
            if neighbor not in open_list:
                heapq.heappush(open_list, (new_cost + heuristic(neighbor, goal), neighbor))
            elif new_cost < neighbor.cost:
                neighbor.cost = new_cost
                neighbor.parent = current_node
```

```
start = Node((0, 0))
```

```
goal = Node((5, 5))
```

```
path = astar(start, goal)
```

```
print(path)
```

Handwritten orange annotations: A bracket on the right side of the code block groups the lines from `new_cost = current_node.cost + 1` to `neighbor.parent = current_node`. Below this bracket, the text "update (node)" is written in orange, with "node" circled. A small star symbol is drawn at the end of the bracket.



A*

Advantages:

- It is optimal search algorithm in terms of heuristics.
- It is one of the best heuristic search techniques.
- It is used to solve complex search problems.
- There is no other optimal algorithm guaranteed to expand fewer nodes than A*.

Disadvantages:

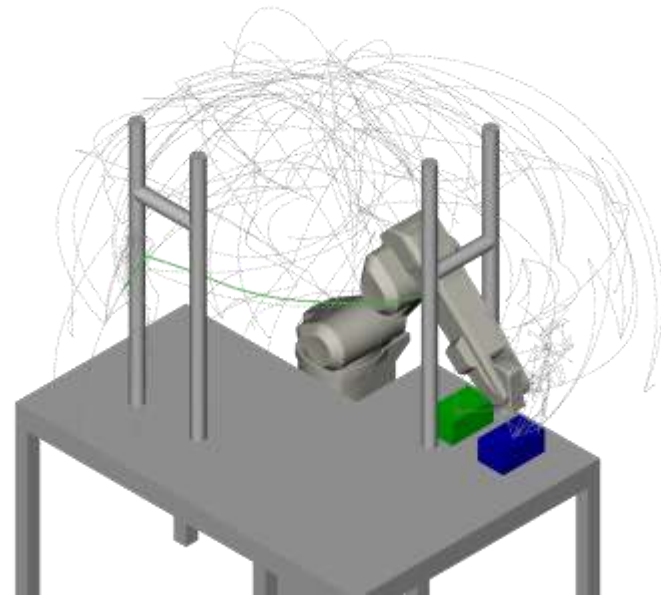
- This algorithm is complete if the branching factor is finite and every action has fixed cost.
- The performance of A* search is dependant on accuracy of heuristic algorithm used to compute the function $h(n)$.

$$f(n) = g(n) + \underline{h(n)}$$
A handwritten diagram of the A* search formula $f(n) = g(n) + h(n)$. The term $f(n)$ is written in blue, $g(n)$ in green, and $h(n)$ in purple. A blue underline is drawn under $h(n)$. Two orange arrows point downwards from the equation: one from $g(n)$ to the first bullet point under 'Disadvantages', and another from $h(n)$ to the second bullet point under 'Disadvantages'.



Today's Agenda

- **What is planning? (~10)**
- **Motion planning in robotic application (~10)**
 - Self-driving, drone, robot arm, humanoids, medical robots, soft robots ...
- **Formulation of robot motion planning**
- **Planning as searching (~25)**
- **Planning as sampling (~25)**
 - PRM, RRT, RRT*



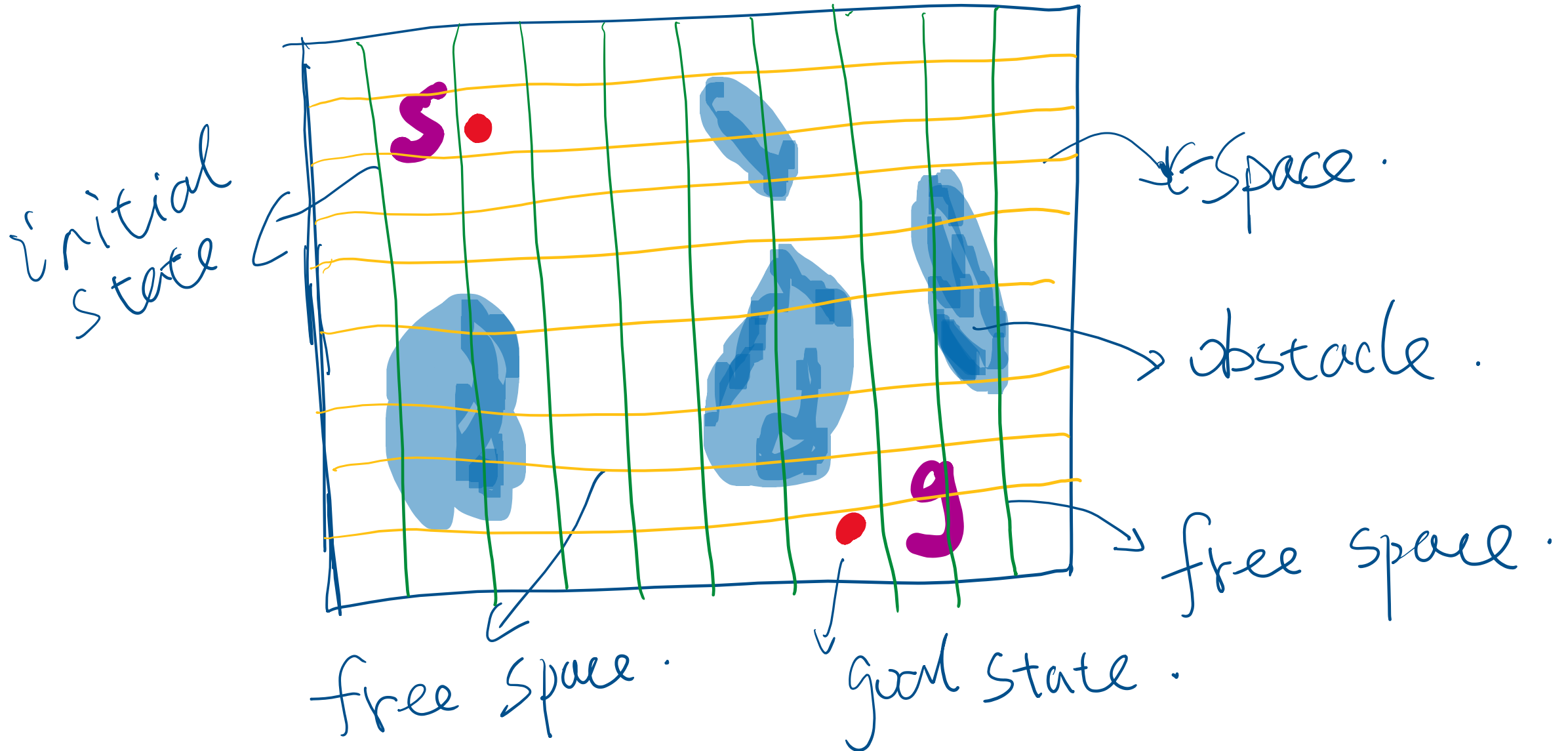


Sampling

- **Completely describing and optimally exploring is too hard in high dimension space**
- **It is not necessary**
- **Limit ourselves to finding a “good” sampling**



Sampling

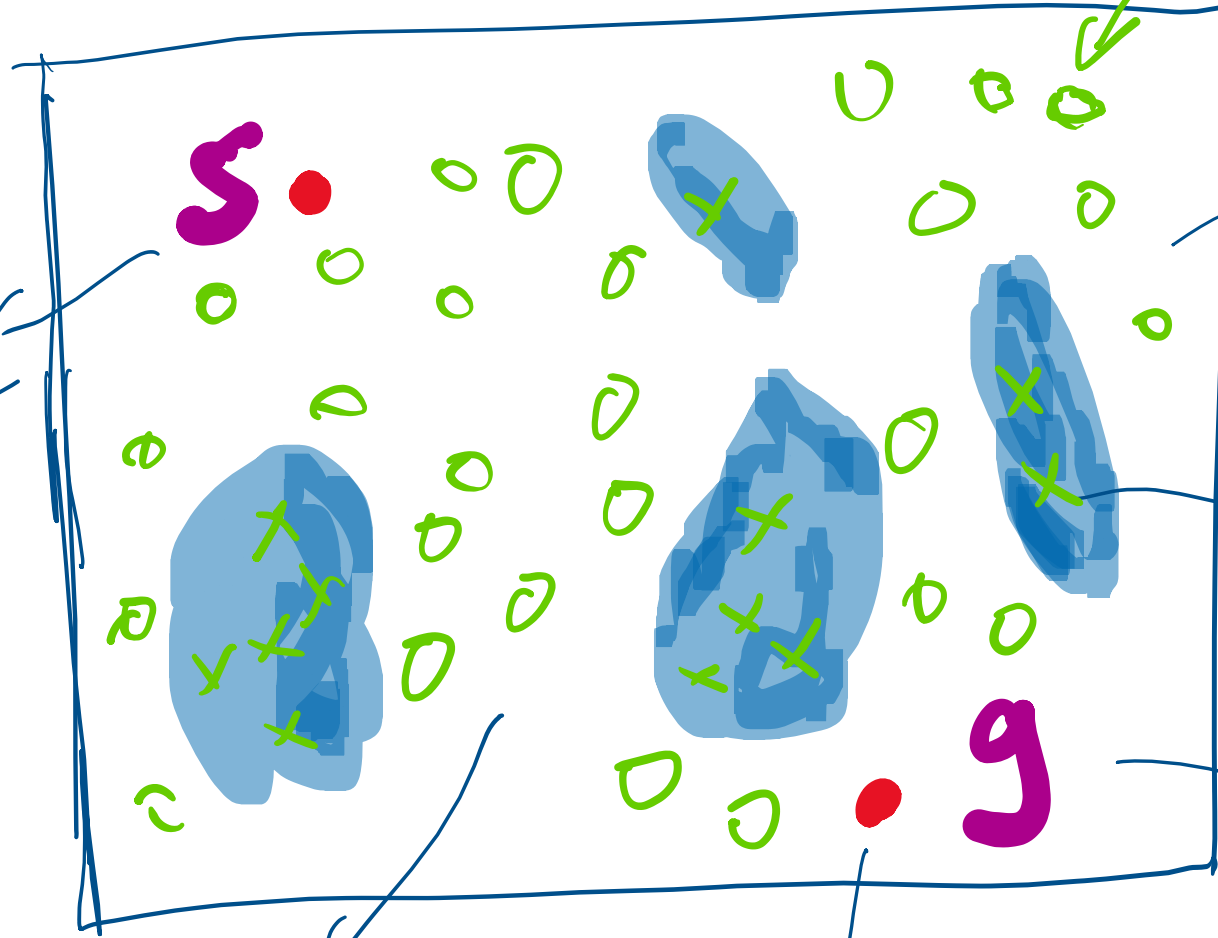




Sampling

sample random locations

Initial State



Space.

obstacle.

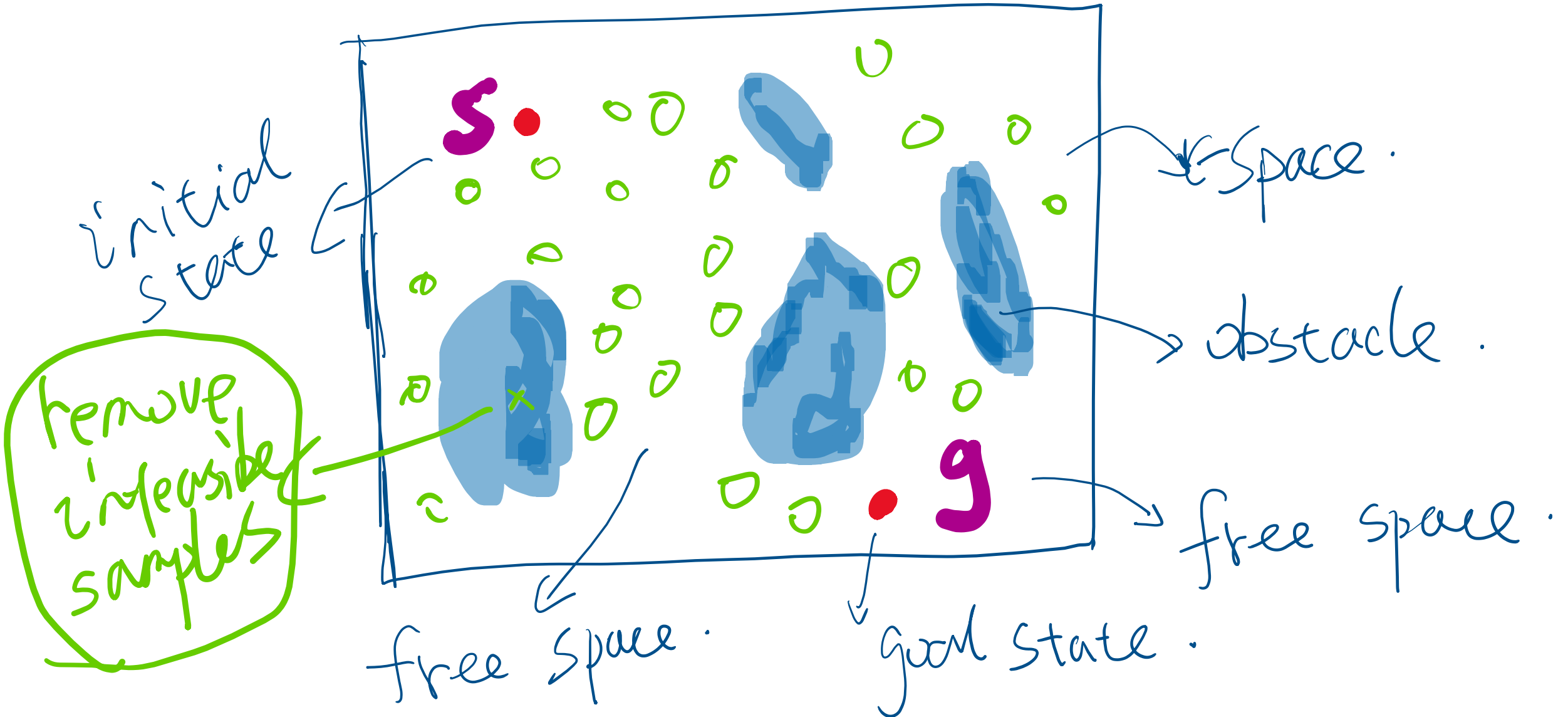
free space.

free space.

Goal State.

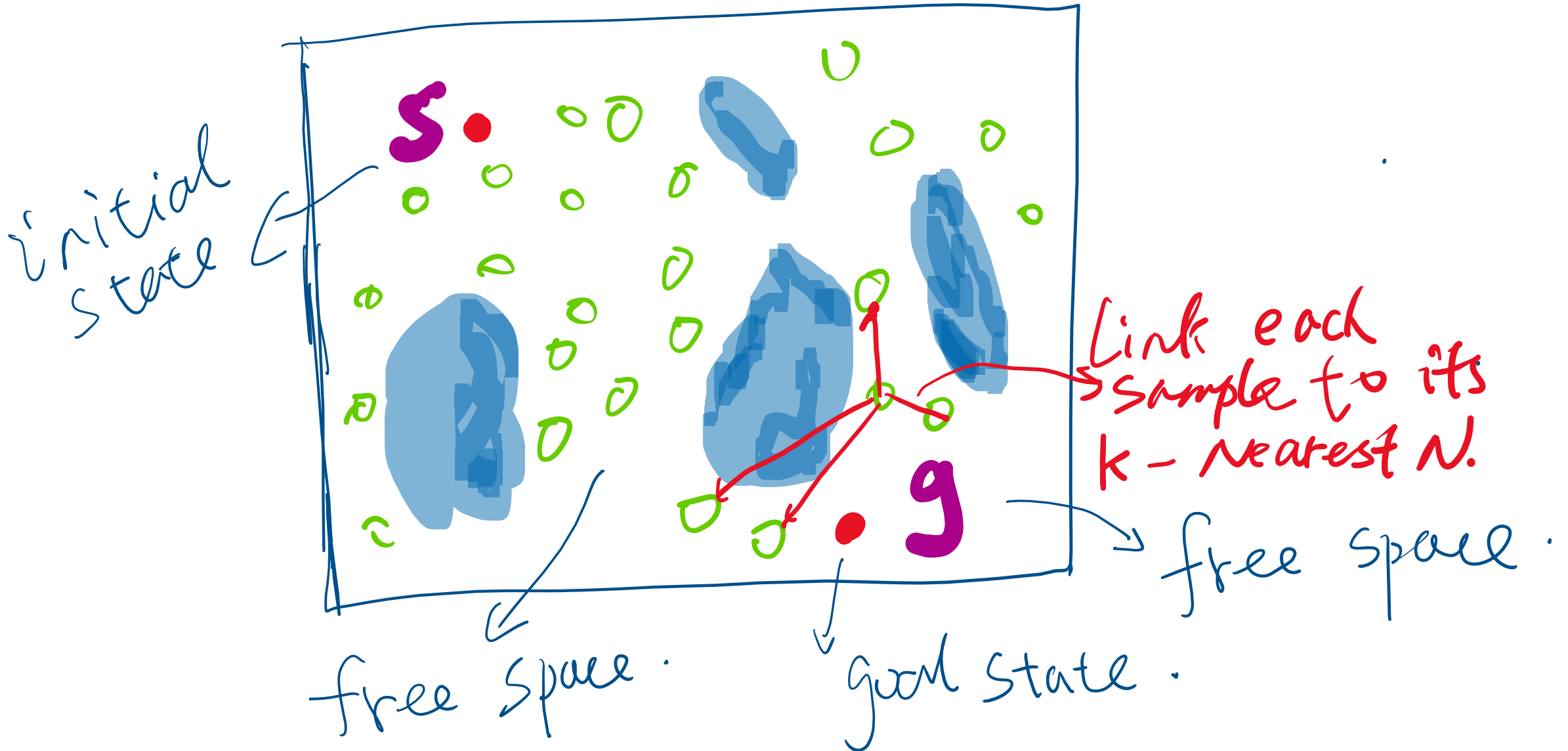


Sampling



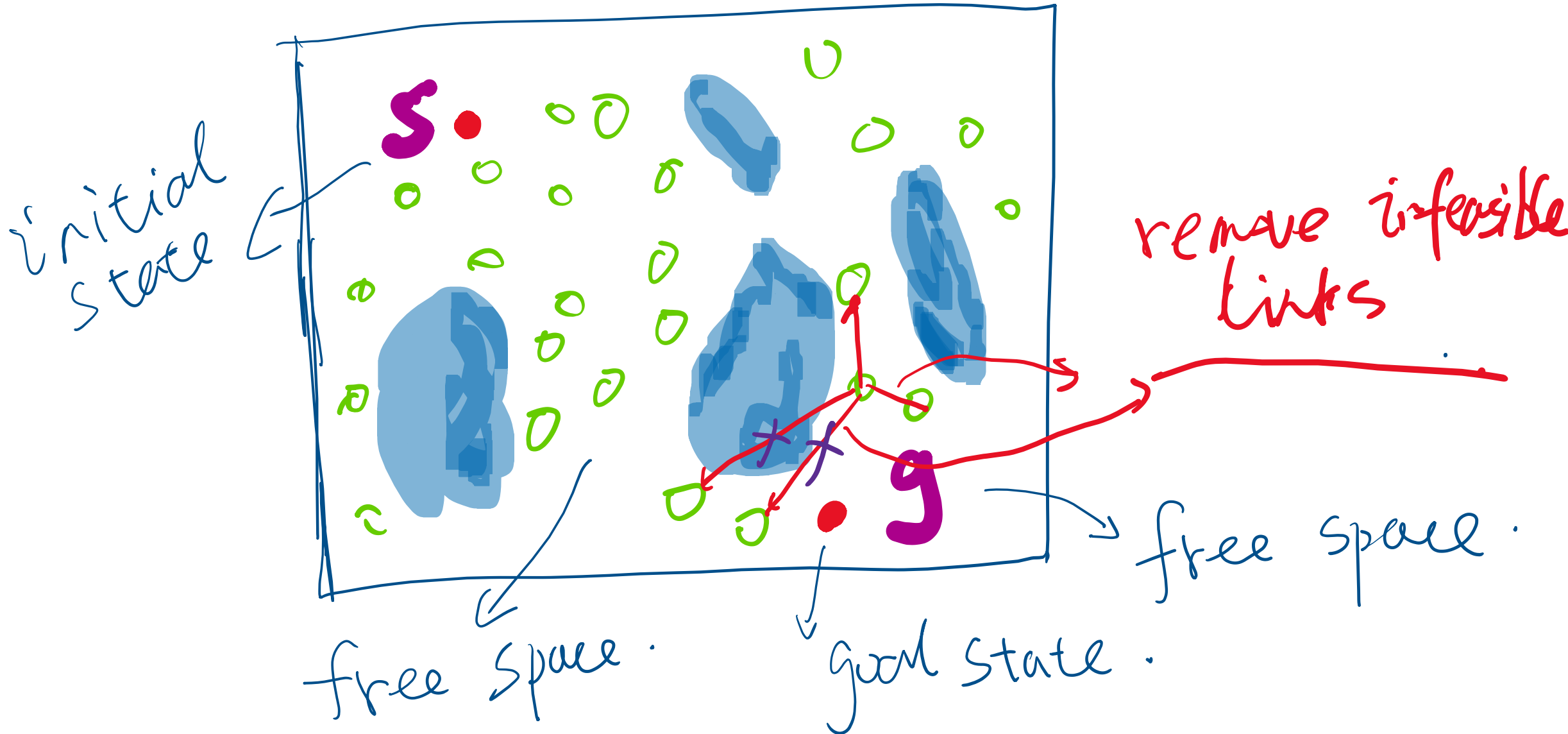


Sampling



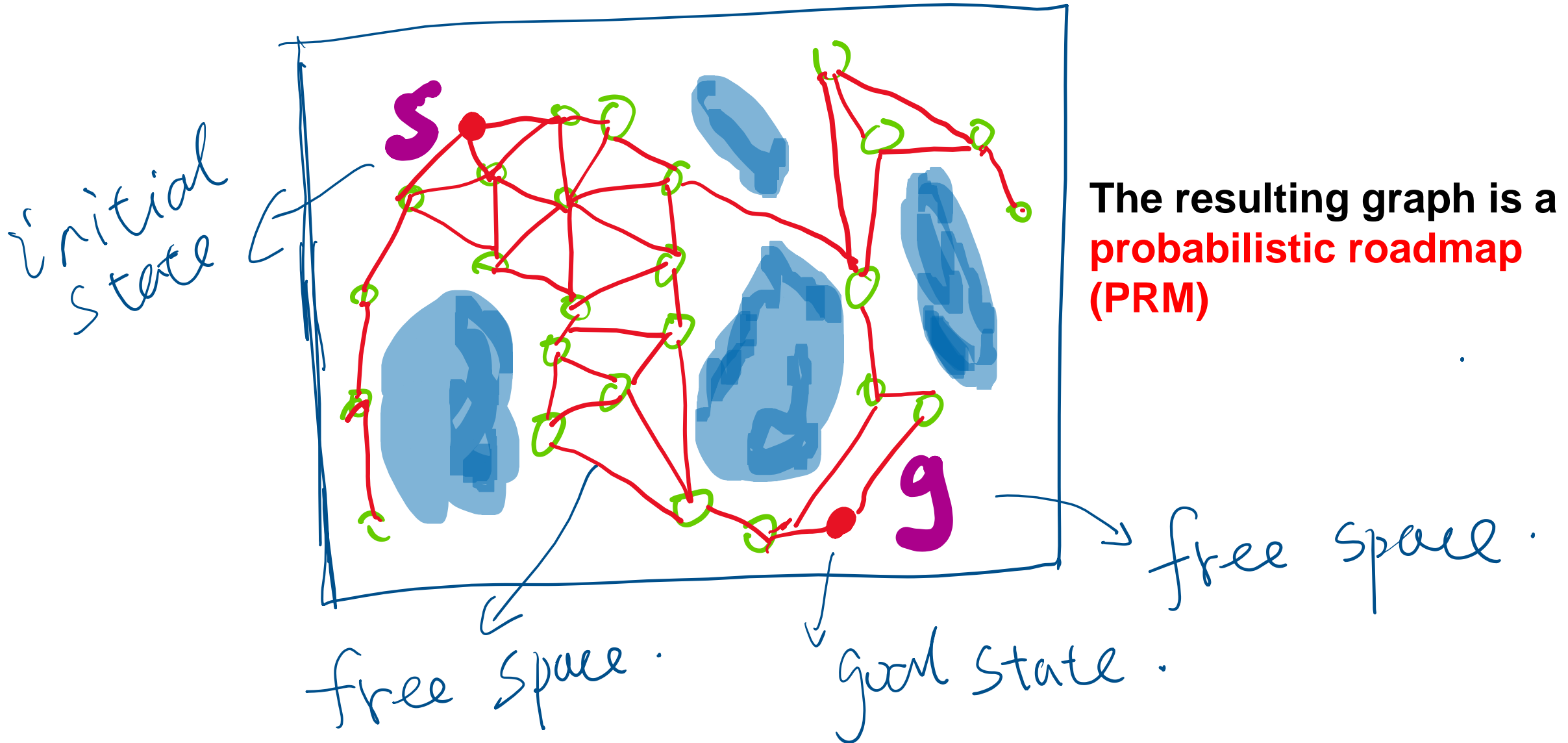


Sampling



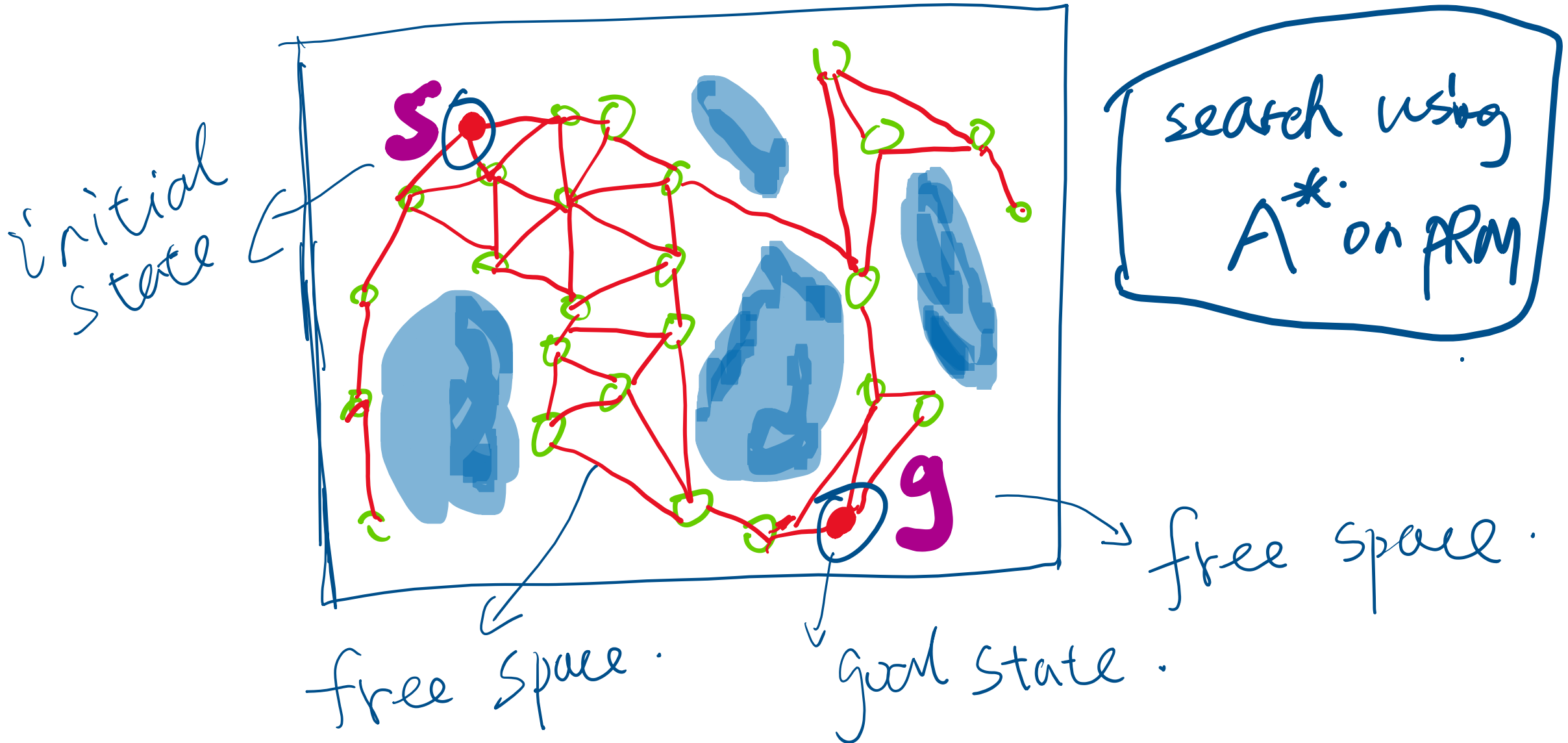


PRM (probabilistic roadmap)





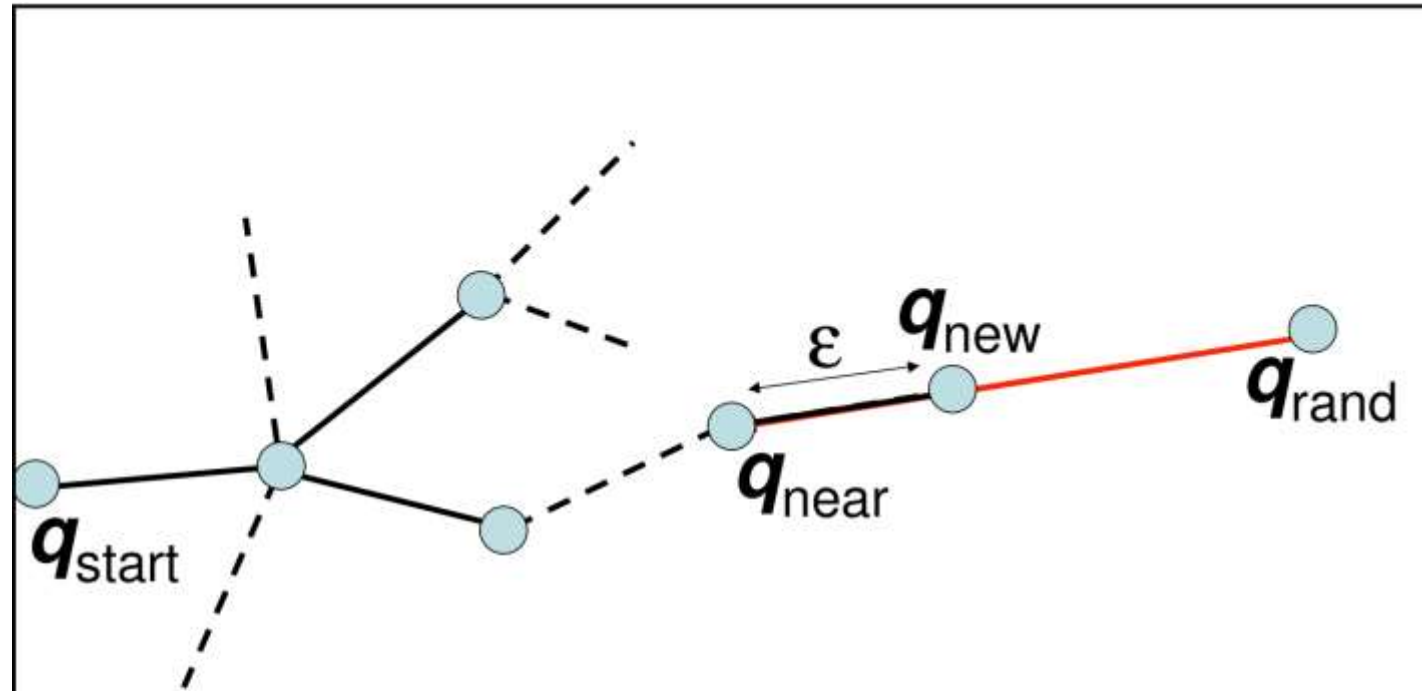
PRM (probabilistic roadmap)





RRT

Rapidly Exploring Random Trees



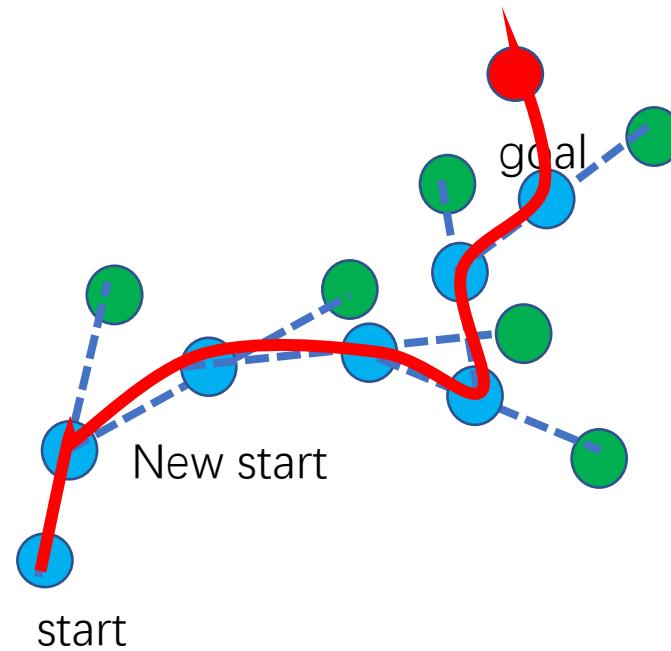
Remarkably, we can find a solution by using *relatively few randomly* sampled points.



RRT

RRT Algorithm ($x_{start}, x_{goal}, step, n$)

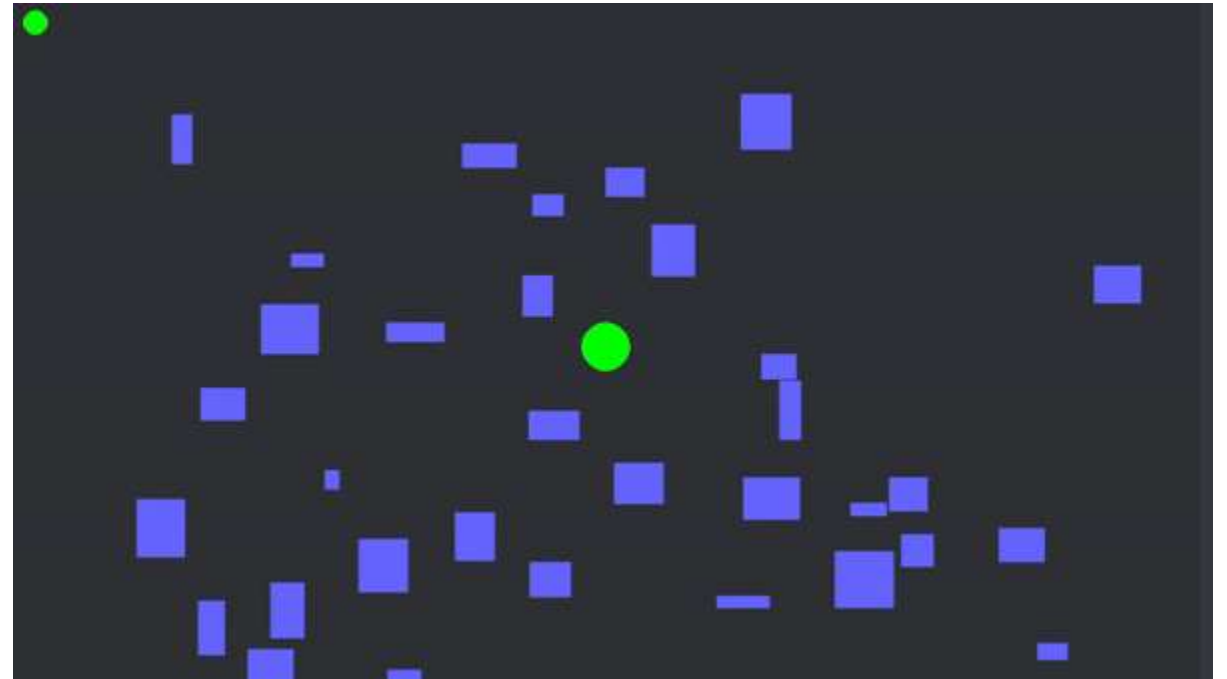
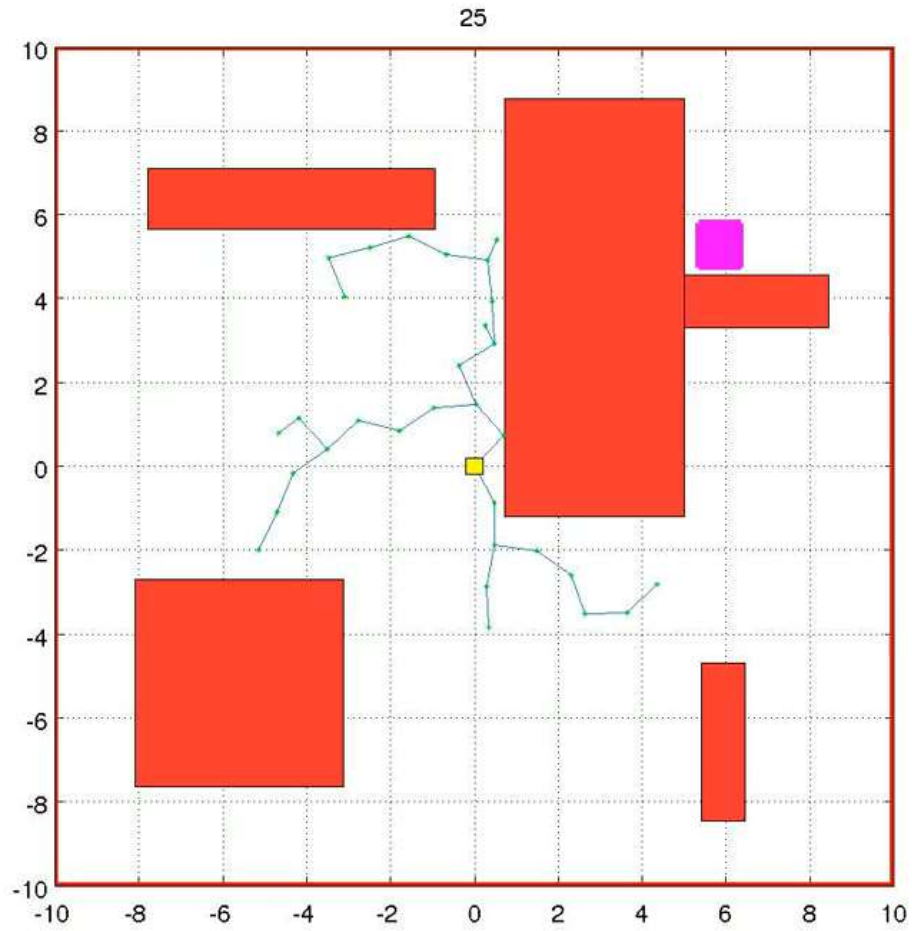
```
1  G.initialize( $x_{start}$ )
2  for  $i = 1$  to  $n$  do
3       $x_{rand} = \text{Sample}()$ 
4       $x_{near} = \text{near}(x_{rand}, G)$ 
5       $x_{new} = \text{steer}(x_{rand}, x_{near}, \text{step\_size})$ 
6      G.add_node( $x_{new}$ )
7      G.add_edge( $x_{new}, x_{near}$ )
8      if  $x_{new} = x_{goal}$ 
9          success()
```



- J-C. Latombe. Robot Motion Planning. Kluwer. 1991.
- S. Lavelle. Planning Algorithms. 2006.
<http://msl.cs.uiuc.edu/planning/>
- H. Choset et al., Principles of Robot Motion: Theory, Algorithms, and Implementations. 2006.

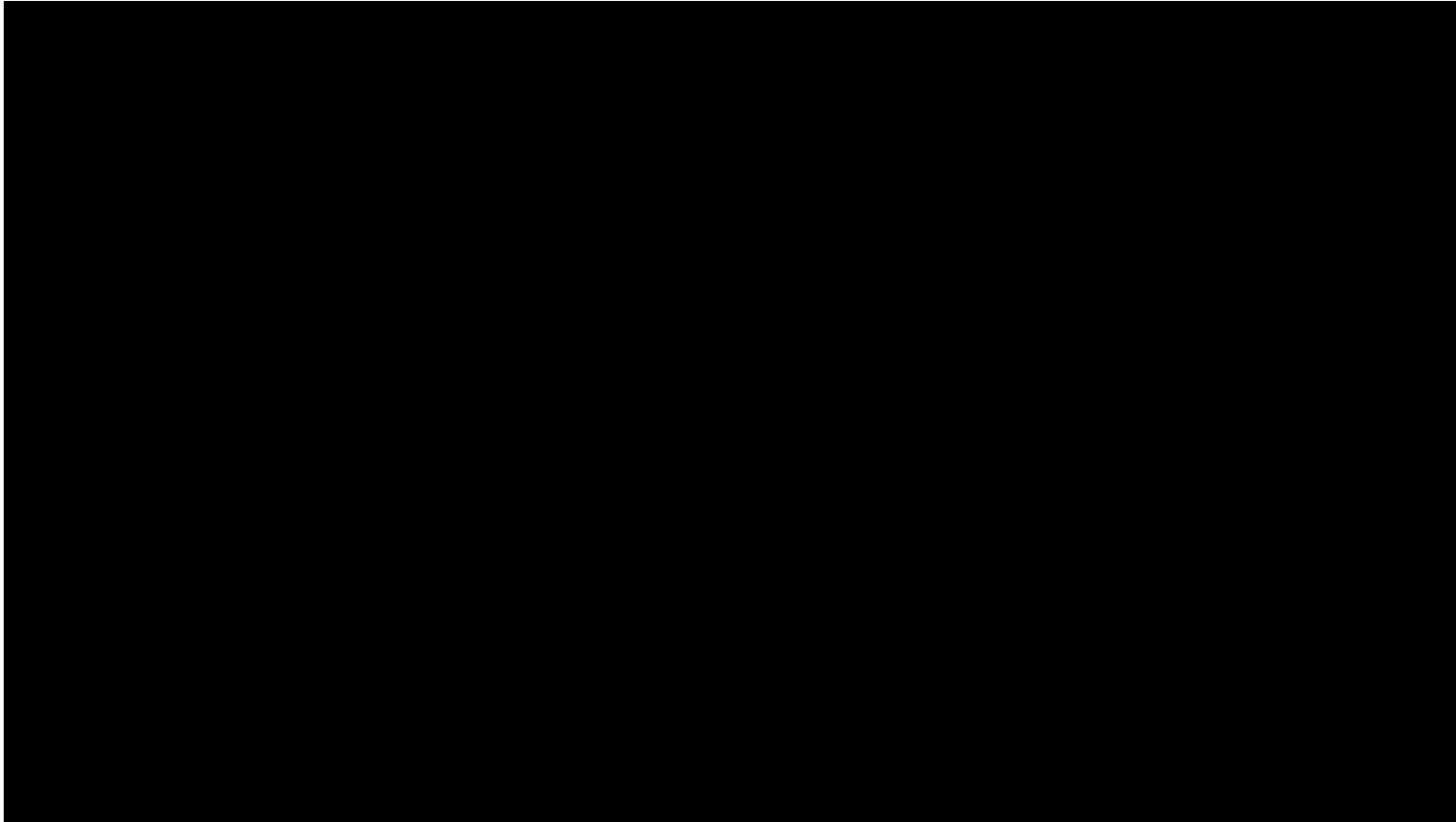


RRT





RRT

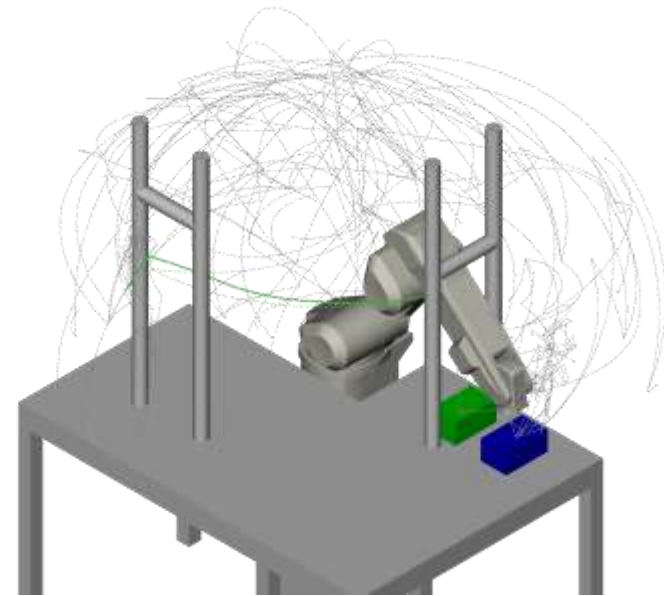


https://www.youtube.com/watch?v=gP6MRe_IHFo&ab_channel=JacksonBernatchez



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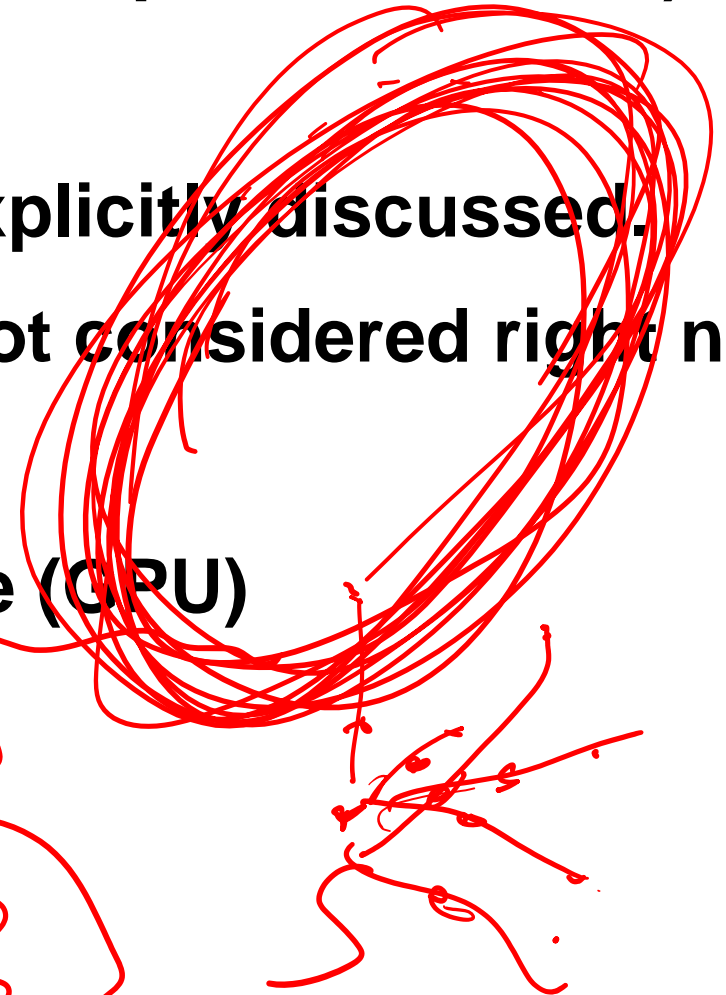


Summary

- Planning is *searching!* (Find a feasible or an optimal solution)
- All the robots are represented as “dots”
- All the obstacles representation are not explicitly discussed.
- The constraints from the robot itself are not considered right now
- The dimensionality (DOF, Task space)
- Robot kinematics and dynamics, hardware (CPU)
- Heuristics design, learning, optimization
- Time complexity, real-time applications

h(h)

M-D
E-D





Goal for this course

- Design: soft hand design **x1**
- Perception: vision, point cloud, tactile, force/torque **x1**
- Planning: sampling-based, optimization-based, learning-based **x3**
- Control: feedback, multi-modal **x2**
- Learning: imitation learning, RL **x2**
- Simulation tool (pybullet, matlab, OpenRAVE, Issac Nvidia, Gazebo)
- **How to get a robot moving!**